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VIBRATION
IN
MACHINERY

AN INTRODUCTION
TO GENERAL PRINCIPLES

BY

W. A. TUPLIN, D.Sc.

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PREFACE

THE importance of acquaintance with the general principles of vibration and of methods of reducing it increases with advance in engineering technique. In general the tendency is to adopt smaller dimensions and lighter constructions in conjunction with higher speeds. Each of these changes tends towards more serious vibration, whereas modern standards of performance require it to be reduced to imperceptible amounts.

A great deal of information has been published from time to time on particular aspects of this many-sided subject, and owing to the mathematical treatment which is necessary for thorough explanation, most of the books and papers dealing with it have a formidable appearance which may deter those who are unwilling or unable to make a very deep study of it.

The present work is intended to give a general review of fundamental principles with the minimum of mathematical complication, to provide some simple formulae for guidance in design, and to form an introduction to the more highly technical publications devoted to special branches of vibration theory.

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MEANINGS OF SYMBOLS

EXCEPT where otherwise specified in the particular instance, the symbols used here have the following meanings—

A = Cross-sectional area (in.^{2}).
B = Harmonic component.
C = Modulus of rigidity (lb./in.^{2}).
D = Diameter (in.).
E = Stretch (Young's) modulus (lb./in.^{2}).
F = Amplitude of force (lb.) or torque (lb. in.).
G = Gear ratio.
H = Damping factor $= \dfrac{\text{energy loss per cycle}}{2 \times \text{maximum strain energy}}$.
I = Moment of inertia (lb. in.^{2}).
J = Polar second moment of area (in.^{4}) about axis perpendicular to plane of area and passing through its centre of gravity.
K = Radius of gyration of section of shaft about axis of shaft (in.).
L = Torque coefficient.
M = Magnification.
N = Frequency (cycles/min.) or speed (revs./min.).
P = Ratio of amplitudes.
Q = Second moment of area (in.^{4}) about axis in plane of area and passing through its centre of gravity.
R = Gear ratio.
S = Damping resistance/Vibration velocity.
T = Ratio of amplitudes.
U = Numeral.
V = Radius of gyration (in.).
W = Weight (lb.).
Y = Order of harmonic component.
Z = Energy input per cycle.
a = Amplitude (in.).
b = Measure of phase.

c = Width of crank web (in.).
d = Diameter (in.).
e = Distance from axis (in.).
f = Force (lb.).
g = Gravitational acceleration (386 in./sec.^{2}).
h = Mean diameter of spring (in.).
i = Moment of inertia (lb. in. sec.^{2}) $= I/386$.
j = Thickness of spring-plate (in.).
k = Radius of gyration of solid of revolution about its axis of generation (in.).
l = Length (in.).
m = Mass (lb. sec.^{2}/in.).
n = Frequency constant, phase velocity, or circular frequency (radians/sec.) $= \pi N/30$.
p = As n, but particularly associated with forced vibration.
q = Number of coils in spring.
r = Radius (in.).
s = Stiffness, linear (lb. in.) or angular (lb. in./radian).
t = Time (sec.).
u = Width of spring-plate (in.).
v = Number of masses.
y = Instantaneous displacement (in.).
z = Density (lb./cu. in.).
α = Phase difference.
δ = Linear displacement (in.).
ε = Phase difference.
θ = Angular displacement (radians).
λ = Amplitude, linear (in.) or angular (radians).
π = 3·1416.
Σ = Summation of all quantities similar to the one indicated.
ω = Angular velocity (radians/sec.).

VIBRATION IN MACHINERY

CHAPTER I

GENERAL PRINCIPLES

VIBRATION in general may be defined as the repetition of a cycle of events. In engineering, perhaps the commonest aspect of vibration is that in which it occurs as a pulsating force. Associated with such a force are pulsating stresses which may determine the endurance of a member, and, if the frequency is within a certain range, pulsating air pressures which are recognized as sound.

In rotary machines, forces of a vibratory nature may be set up by unbalance in a rotating member. In machines which combine rotation and reciprocation there are usually alternating forces applied to or by the reciprocating parts and corresponding fluctuations of torque in the crankshaft. The forces may be transmitted through the foundations and the earth to places where their effects are particularly objectionable. In certain circumstances the fluctuating torques may lead to failure of the crankshaft itself apart from any possibility of transmission of vibration elsewhere.

Frequency. The complete sequence of repeated events in any vibration is called a *cycle*. The *frequency* of the vibration is the number of cycles occurring in unit time. The time occupied by one cycle is known as the *period*.

Vibration frequencies which are mechanically important in engineering may be said to range from about 100 to 20 000 cycles/min. Audible frequencies lie roughly between 2 000 and 250 000 cycles/min.

Amplitude. The amplitude of a vibration is a measure of the range over which variation takes place during one cycle. There is not at present uniformity in defining amplitude; in the following it is taken to refer to *half* the range. Thus, if the stress at a certain point varies over a regular cycle between 4 000 lb./in.^{2} and 16 000 lb./in.^{2}, the amplitude of stress variation is half the difference between these figures, i.e. 6 000 lb./in.^{2} An alternating stress of amplitude 6 000 lb./in.^{2} is regarded as superimposed on a mean stress of 10 000 lb./in.^{2}.

Units. In all numerical work in connexion with vibrations special care is necessary in stating the units of measurement. In the following, these units are used—

Weight	pounds
Force	pounds
Length	inches
Time (in general)	seconds
Frequency	cycles per minute

Simple Harmonic Vibrations. If a point P (Fig. 1) moves with constant speed round the circumference of a circle, the foot Q of the perpendicular dropped from the point on to a diameter AB moves backwards and forwards between A and B, and its motion is an example of vibration. One cycle is completed each time P moves round the circle. Hence the frequency of the vibration is the frequency of rotation of OP. The amplitude of the vibration is equal to the radius of the circle.

The application of elementary kinematics shows that the acceleration of Q is always directed towards O and is proportional to OQ. This characteristic suffices to define Q's movement as a *simple harmonic* vibration. Mathematically this type of vibration is the simplest of all, and fortunately many vibrations which occur in engineering are either simple harmonic or can be treated as combinations of simple harmonic vibrations.

Vibration about a Moving Mean Position. In Fig. 1 the mean position of the vibrating point Q is the centre O, assumed to be stationary. An example approximating to this is a pendulum's motion, which is substantially horizontal. The mean position of the moving mass is stationary, vertically below the point of suspension.

If now the point of suspension be given a uniform horizontal velocity, the horizontal velocity of the vibrating mass goes through a regular cycle, and its mean value is the velocity of the support. Thus the mean position of the vibrating mass has the velocity of the support, and vibration takes place about a uniformly moving mean position. In investigating such vibrations, the uniform motion of the mean position may be ignored.

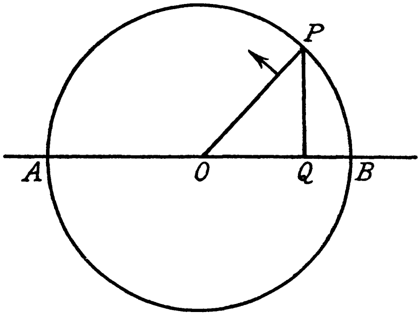


Fig. 1. Simple Harmonic Motion derived from Uniform Circular Motion

The commonest case in which this point arises is that of torsional vibration where rotating parts have angular vibrations, with the result that their angular velocities are continuously varying. The vibrations of any particular member may be regarded as taking place about a mean angular position rotating uniformly at the mean speed of that member.

General Form of Vibration. It can be shown that any quantity whose value varies continuously over a regular cycle of frequency N may be represented by an expression of the form—

$$a_0 + a_1 \sin (nt + b_1) + a_2 \sin (2nt + b_2) + a_3 \sin (3nt + b_3) \ldots \text{etc.} \quad (1)$$

where a_0, a_1, a_2, etc., and b_1, b_2, etc., are constants, t represents time, and $n = 2\pi N$.

For simple harmonic variation with mean value zero, $a_0 = 0$ and $a_2 = a_3 = \text{etc.} = 0$, leaving only the term $a_1 \sin (nt + b_1)$, where $a_1 =$ amplitude, $n =$ frequency constant or phase velocity, and $b_1 =$ constant depending on the instant from which time is measured.

The terms deleted are of the same form as this, but they correspond to frequencies which are multiples of N. Hence any periodic variation can be represented as the sum of a number of simple harmonic vibrations having frequencies which are integral multiples of the fundamental frequency. This is of extreme importance in analysing vibrations occurring in machinery.

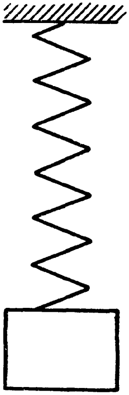


Fig. 2. Vibrating System with Single Elastic Element

Free Vibrations. Any material body which possesses mass and elasticity (and this applies to all) is capable of *natural vibration* of at least one kind. In Fig. 2 is represented a weight carried by a helical spring. In this the spring possesses elasticity and little mass, while the weight is oppositely endowed. This assembly is chosen for discussion because these characteristics are distinct; in general, a machine part or structural member has both elasticity and mass.

A vertical displacement of the weight from its equilibrium position changes the length of the spring and causes the axial load in it to differ from the weight by an amount proportional to the displacement. This difference is a *restoring force* directed towards the equilibrium position, and the acceleration of the weight in that direction is thus proportional to the displacement.

If there were no restraining influences, the weight would commence a simple harmonic vibration and would continue indefinitely in that state with constant amplitude. Actually there are resistances which absorb energy and thus cause the amplitude to diminish gradually until all vibration ceases. Such vibrations are said to be *damped*. Vibrations which occur without being accompanied by any external influence that tends to maintain them are called *free* or *natural* vibrations. So far as the definition is concerned, they may or may not be damped. In practice there is always damping of one kind or another.

Ignoring for the moment these damping influences, the free vibration of the system shown in Fig. 2 is simple harmonic. Since in practice a small displacement from a position of stable equilibrium usually induces a restoring force proportional to the displacement, it is very common to find that natural vibrations are simple harmonic.

If we assume that the free vibration of the weight in Fig. 2 is simple harmonic, we may write—

$$\text{Displacement} = y = a \sin nt$$

where a is a constant.

We have, after double differentiation with respect to t—

$$\text{Acceleration} = \frac{d^2y}{dt^2} = -n^2 a \sin nt = -n^2 y$$

Hence, if the assumption of simple harmonic motion is to be justified, we must have—

$$\text{Acceleration/Displacement} = -n^2 = \text{constant}$$

and if this is so we may write (suppressing the negative sign, which is unimportant here)—

$$\text{Natural frequency} = N = \frac{n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} \quad (2)$$

The quantity n is called the *frequency constant* or phase velocity or circular frequency.

If the stiffness of the spring is a constant s, the restoring force for any displacement y from the equilibrium position is sy. Hence—

$$\frac{\text{Acceleration}}{\text{Displacement}} = \frac{sy/\text{Mass}}{y} = \frac{s}{\text{Mass}}$$

which is a constant, and the original assumption is justified. Therefore the natural frequency can be determined from the stiffness of the spring and the suspended mass, and is given by—

$$N = \frac{1}{2\pi}\sqrt{\frac{s}{\text{Mass}}} \quad . \quad . \quad . \quad (3)$$

Also $$dy/dt = na\cos nt$$

and therefore the maximum velocity of the weight is na, this occurring when $\cos nt = 1$. At that instant $\sin nt = 0$, $y = 0$, and the strain energy of vibration is zero.

The maximum kinetic energy is therefore $\frac{1}{2}$ Mass × $(na)^2$, and this represents the total energy of vibration.

The maximum strain energy occurs when $y = a$, and is then equal to $\frac{1}{2}a^2s$. This also represents the total energy of vibration, since at that instant the kinetic energy is zero. Equating the two maxima, it is found that $n^2 = (s/\text{Mass})$ and as before—

$$N = \frac{n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{\text{Mass}}}$$

Using the inch and the second as units, Mass = Weight/386, and since Weight/s = static deflection, substitution in the formula for N gives—

$$N = \frac{1}{2\pi}\sqrt{\left(\frac{386}{\text{Static Deflection}}\right)} = \frac{3{\cdot}13}{\sqrt{(\text{Static Deflection})}} \text{ cycles/sec.}$$

$$= \frac{188}{\sqrt{(\text{Static Deflection})}} \text{ cycles/min.} \quad . \quad . \quad (4)$$

This principle of equating maximum kinetic energy to maximum strain energy is a useful one in calculating natural frequencies of complicated vibrating systems.

Stiffness. The natural frequency of a single-mass system may be determined from equation (4), the static deflection used being the movement of the point of attachment of the mass to the elastic element under a force, equal to the weight of the mass, acting in the direction of the vibrating movement. If the vibration is torsional, the static deflection is the angular movement under a torque that would produce an acceleration of 386 rad./sec.^{2} in the mass.

In general, however, it is necessary to determine the "stiffness" of the elastic element, i.e. the force required to produce unit linear displacement or the torque required to produce unit angular displacement. If the physical properties of the elastic material are known, the stiffness can usually be calculated from the dimensions of the elastic element.

Table I (page 96) gives formulae for displacement under unit force or torque for a number of typical geometrical forms representative of machine design. The stiffness is the reciprocal of this displacement. Table II (page 99) gives formulae for moment of inertia of certain solids.

Damping. It has been shown that if no external forces are applied to the spring-and-mass assembly the motion of the mass continues indefinitely, the displacement, defined by $y = a \sin nt$, being represented by the upper curve in Fig. 3.

In practice, natural vibrations gradually diminish in amplitude as the result of energy-dissipating effects such as friction, mechanical hysteresis, viscous drag of fluids, and air resistance. In the particular case considered, the internal friction in the material of the spring and the air resistance opposing the motion of the weight play the most important parts, but the rate of reduction of amplitude is slow (it may, for example, take thirty vibrations to halve the amplitude).

If the weight be caused to vibrate in a vessel containing a viscous fluid such as oil, the amplitude diminishes rapidly, and the variation of displacement with time is represented by a curve such as that shown by the lower part of Fig. 3.

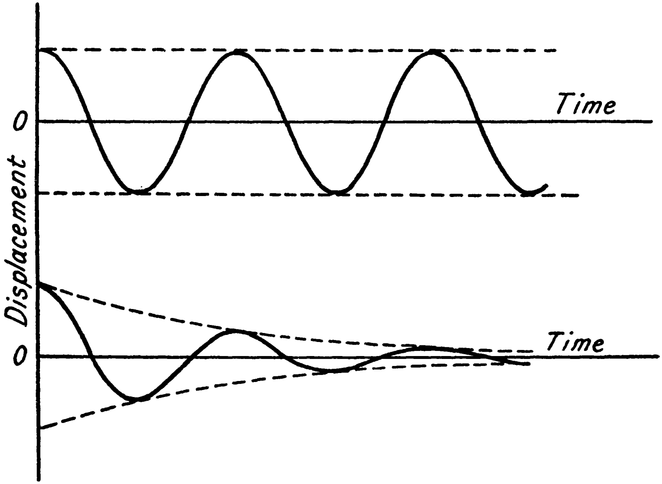


FIG. 3. DISPLACEMENT/TIME CURVES FOR UNDAMPED AND DAMPED HARMONIC MOTION

Such a vibration is said to be *damped*, and the various resistances which combine to oppose the vibration are called *damping influences.* If they are very strong they may prevent vibration altogether. For example, if the weight be placed in a vessel of very thick oil and be given a displacement from the equilibrium position, it will find its way slowly back to that position without overshooting it and therefore without any oscillatory motion.

The smallest amount of damping which will prohibit vibration is called *critical damping.* It permits the mass to return as quickly as possible to its equilibrium position without overshooting the mark. This is important in an instrument (e.g. ammeter) which has to record accurately sudden changes in the value of a quantity, and an instrument which has the critical amount of damping is said to be *dead beat.* Damping in such cases is provided by a dashpot using air as the resisting fluid or by attaching to the moving body an armature lying between the poles of a powerful magnet.

It is important to note that normal damping influences, even though they may cause the vibration to die away rapidly, have only a small effect on the natural frequency of the system.

Forced Vibration. A vibration which is maintained by one or more external oscillatory influences is called a *forced vibration.*

If the suspended mass shown in Fig. 2 be subjected to an alternating force acting along the axis of the spring, the weight will move up and down with the same frequency as the force. If the weight is immersed in oil, the viscous damping arising therefrom will reduce the amplitude of the weight's vibration. The actual value of the amplitude depends on the magnitude and frequency of the external force, and upon the degree of damping. Fig. 4 indicates the nature of the relation between amplitude and frequency on the assumption that the external force is of simple harmonic character, i.e. that its value at any instant is represented by an expression of the form $f = F \sin pt$, where F and p are constants. Even if this is not so, the force's variation with time can be represented by a compound harmonic expression similar to (1), and its total effect determined as the sum of those of a number of simple harmonic forces. The dotted curve in Fig. 4 refers to the undamped condition.

Since the tension in the spring is proportional to the displacement of the mass from its equilibrium position, the amplitude of the tension in the spring is proportional to the amplitude of displacement of the mass. Consequently amplitude of spring tension may be used as a measure of the magnitude of the forced vibration. Since the amplitude of the external force will obviously have an influence on the severity of the vibration, the ratio

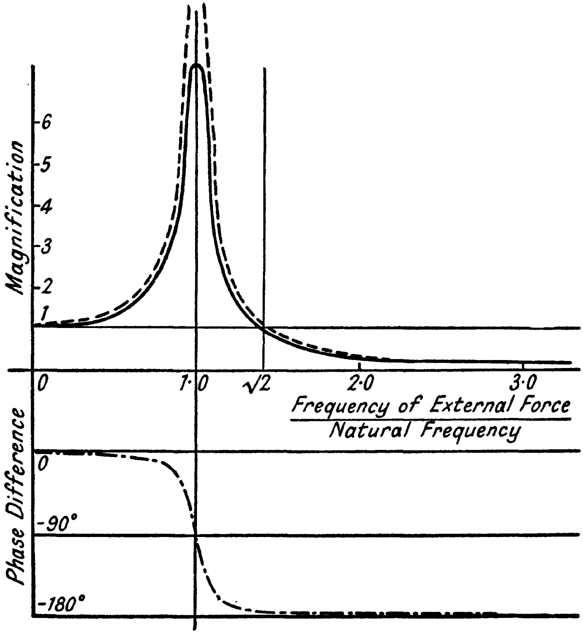


FIG. 4. VARIATION OF MAGNIFICATION AND PHASE DIFFERENCE WITH FREQUENCY

$$\frac{\text{amplitude of spring tension}}{\text{amplitude of external force}}$$

is a measure of the dynamic effect of the spring-mounting of the mass. This quantity is called the *magnification* or the *dynamic magnifier*.

Similarly the frequency of the external force may be expressed as a multiple of the natural frequency of the system.

The important points which emerge from an inspection of Fig. 4 are these—

1. Damping diminishes the magnification at all frequencies.
2. The greatest magnification occurs when the frequency of the external force is equal to the natural frequency. This is the condition of *resonance.*
3. Apart from damping, the magnification would be infinite at resonance.
4. If the frequency of the external force is a very small fraction of the natural frequency, the magnification is almost unity.
5. If the frequency of the external force is a large multiple of the natural frequency, the magnification is almost zero.
6. Damping has a negligible effect on the magnification except in the neighbourhood of the condition of resonance.

Any damping resistance which increases with the vibration velocity is capable of setting a limit to the amplitude at resonance. (Damping resistance proportional to vibration velocity is a particular case.) The reason is that any such damping resistance causes energy dissipation proportional to the displacement amplitude raised to a higher power than unity, whereas the energy input by an exciting force or torque of constant amplitude is proportional to the amplitude of displacement of its point of application. Consequently energy input and energy dissipated are equalized at one particular value of the displacement amplitude.

Any damping resistance (e.g. due to hysteresis) which causes energy dissipation proportional to a power of the amplitude higher than the first can set a limit to the amplitude at resonance.

In many practical cases the vibration-exciting forces are so small that the problem of avoiding serious vibration becomes simply the problem of so arranging the natural frequencies as to avoid the condition of resonance at normal operating speeds.

Magnification and Phase Difference. In general, the vibratory motion of the suspended mass is not in step with the variation of the force applied to it. If the value of the external force is represented by $f = F \sin pt$, that of the force in the spring is given by—

$$f_s = MF \sin (pt - \alpha)$$

Here M is the magnification factor, since it is the ratio of the amplitude MF of the force in the spring to the amplitude F of the external force.

The quantity α is the phase difference between the two forces. Each has the same frequency $p/2\pi$, since in each case the value is repeated if t is increased by $2\pi/p$, but the instants at which the forces become zero are separated by a time α/p.

It may be shown that—

$$M = \frac{1}{\sqrt{[\{1 - (p/n)^2\}^2 + (p/n)^2 \cdot 1/M_0{}^2]}} \quad . \quad (5)$$

and

$$\tan \alpha = \frac{1}{M_0} \cdot \frac{1}{1 - (p/n)^2} \cdot \frac{p}{n} \quad . \quad . \quad (6)$$

where n = frequency constant for natural vibration of undamped system,

p = frequency constant for external force,

M_0 = magnification when $p = n$.

The quantity M_0 is a measure of the damping in the vibrating system. It is also the value of the magnification when $p = n$, i.e. when the frequency of the external torque is equal to the natural frequency of the undamped system. For all practical purposes this is the maximum magnification, applying to the condition of resonance.

The expressions given for M and $\tan \alpha$ apply strictly only when the damping effect is equivalent to a resisting force proportional to the vibration velocity, this case being the simplest from the point of view of mathematical analysis. It is found in practice that the expression for M applies with satisfactory accuracy, even though the damping effect cannot be shown directly to be of that particular mathematical nature.

It may be shown that—

$$M_0 = 2\pi \left[\frac{\text{maximum strain energy of system}}{\text{energy loss per cycle by damping}}\right]$$

and this gives the value of M_0 when the magnitude of the damping loss is known for any particular vibration amplitude. For most practical cases it is found that the value of the expression in the square bracket is constant, irrespective of the amplitude of vibration, and in that case the magnification is a constant for the system. If the damping is confined to internal friction (mechanical hysteresis) in stressed metal parts, M_0 may be of the order of 600.

Taking a large value such as this and tracing the variation of tan α from (6) as p/n is varied, it is found that α is nearly zero or nearly 180° except in the neighbourhood of $p = n$. In words, the force exerted by the weight on the spring lags slightly behind the external force on the weight when the frequency is low, and is the maximum distance out of step when the frequency is high.

In the resonance condition, $\alpha = 90°$ and the forces are said to be *in quadrature*. When the external force is at its maximum value, the weight is at its mid-position.

When the frequency of the applied force appreciably exceeds the natural frequency, the lag becomes nearly 180°. In this condition the weight is nearly in its uppermost position when the external force is at its maximum upwards value. The internal and external forces are almost exactly out of step, and are said to be *antiphased.*

This variation in phase difference is expressed by the chain-dotted curve in Fig. 4. The most important feature in practice is the sudden change, on passing through the resonance condition, from "in step" to "out of step." The greater the amount of damping in the system, the more gradual does this change become.

Damping Influences. Every actual vibrating system is subjected to damping influences which set a limit to the amplitude that can arise at resonance, but they may not be strong enough to avoid the development of objectionably high stresses. The most common damping influences are—

1. *Mechanical Hysteresis.* Friction between the relatively moving molecules of the elastic element in the vibrating system causes dissipation of energy in the form of heat, and therefore maintenance of a constant amplitude of vibration involves a continuous input of work. The phenomenon of internal friction is called *mechanical hysteresis*, or simply *hysteresis*. The magnitude of the hysteresis is measured by the ratio (energy lost per cycle)/(maximum strain energy), and for metals under tensile or compressive stress below the elastic limit it is of the order of 0·01. For organic materials it is usually much higher. For rubber it may be as high as 0·5 in loaded members shaped so as to permit lateral expansion.

2. *Air Resistance.* Movement of the exposed surfaces of the vibrating system is opposed by air resistance. In most practical cases, however, this effect is negligible, owing to the low density and viscosity of the air and the low velocity of vibration. Substitution of, say, oil for air, were it practicable, would multiply the damping effect by some hundreds.

3. *Variation of Resistance with Vibration Velocity.* Torsional vibrations are frequently damped by variation in the resisting torque of the driven machine. In many types of rotary machine (e.g. centrifugal pump electrical generator) the resisting torque is proportional

to the angular velocity raised to some positive power (very often approximately 2). Consequently, velocity variations produced by torsional vibration about a uniformly rotating mean position introduce torque variations, and these tend to oppose the vibration. This may be a very powerful damping effect.

4. *Viscosity in Oil Films.* Where vibration causes journals to move laterally in their bearings, the resistance offered by oil-film viscosity to such changes in position causes dissipation of energy and is a damping influence.

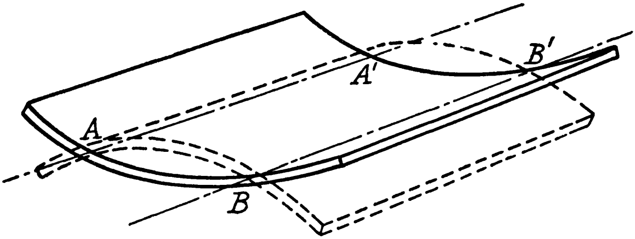


Fig. 5. Nodal Lines in Flexurally Vibrating Plate

In particular cases there may be damping influences besides the four sorts mentioned above, but they are usually unimportant. It may be noted that sliding friction that is independent of the sliding velocity cannot prevent the attainment of infinite amplitude at resonance and is therefore of negligible importance in that condition.

Nodes. In any system in a state of natural vibration there is at least one point at which the amplitude of displacement is zero. Such points are called *nodes.* For example, the upper end of the spring in Fig. 2 is the only node in the system.

A point at which the amplitude of displacement is greater than at points immediately on each side of it is called an *antinode.* In Fig. 2 the suspended mass is the only antinode.

A vibrating body may often be imagined as divided up into an infinite number of similar vibrating elements. The various nodes in these elements lie on *nodal lines*. Thus in Fig. 5 the end view of the plate is a narrow rectangle which, when vibrating flexurally in its own plane, may have nodes A and B. The corresponding nodes in other imaginary rectangular sections of the plate lie on the nodal lines AA' and BB'.

CHAPTER II

MULTI-MASS SYSTEMS

A VIBRATORY system may have (in general) any number (v) of masses, and any number of elastic elements up to $\frac{1}{2}v(v-1)$. Strictly speaking, every system may be regarded as composed of an infinite number of infinitely small masses, since every part of the material system possesses both mass and elasticity.

Any system which has more than one elastic element has more than one mode of natural vibration. Important facts regarding the natural vibration forms of multi-mass systems are these—

1. Different modes of vibration give different natural frequencies.
2. If all the vibration forms are arranged in ascending order of natural frequency, they are also in ascending order of number of nodes.
3. The number of vibration forms is equal to the number of elastic elements in the system.
4. The vibration form of lowest frequency contains a single node.
5. The vibration form of highest frequency contains one node in each elastic element.

Determination of Natural Frequencies of Multi-mass Systems. The principle of the method of determining the natural frequencies of vibration of a "straight-line" multi-mass system such as that shown in Fig. 6 is simple. It is, however, a trial-and-error process, and may therefore occupy some considerable time. In Fig. 6 the rectangles represent masses and the zigzag lines elastic elements.

An assumed value n is taken as the frequency constant for a natural vibration. The amplitude of vibration of mass No. 1 (m_1) is taken to be unity. Then the amplitude of the force acting on mass No. 1 is $n^2 \times 1 \times m_1 = n^2 m_1$. This is the amplitude of the force transmitted by spring No. 1. If its stiffness is s_1, the maximum stretch in that spring is $n^2 m_1/s_1$, and therefore the amplitude at mass No. 2 is equal to $1 - (n^2 m_1/s_1)$. Hence the amplitude of the total force on mass No. 2 is $n^2 \, [1 - (n^2 m_1/s_1)]m_2$. But the force in spring No. 1 is $n^2 m_1$, and therefore—

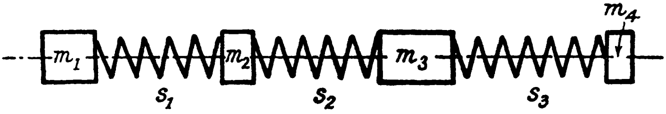


FIG. 6. FOUR-MASS SYSTEM

$$\begin{array}{c}\text{Amplitude of force}\\ \text{in spring No. 2}\end{array} = n^2\left(1 - \frac{n^2 m_1}{s_1}\right)m_2 - n^2 m_1$$

This procedure is repeated until it is found that a certain external force would require to be applied to mass No. 4 (additionally to that exerted by spring No. 3) in order to give it the calculated amplitude of vibration.

As a natural vibration is one which proceeds without such external force, the problem becomes that of finding values of n which make the external force on the last mass equal to zero. An advantage of this procedure is that it gives automatically the amplitude of displacement of each mass expressed as a multiple of that of mass No. 1. These relative amplitudes give the *normal elastic curve*, which is discussed in the next paragraph. The amplitudes of displacement at points in any elastic element which is regarded as massless vary uniformly between the amplitudes at the ends of the element.

The calculations are most conveniently made in tabular form.

Normal Elastic Curve. If, on a diagrammatical representation of the system wherein elastic elements are indicated by adjacent segments of a straight line, ordinates are erected of lengths proportional to the amplitudes of displacement at their feet during a free vibration, the curve obtained is called the *normal elastic curve* for that particular type of free vibration.

In a condition of resonance the amplitudes of forces acting within the system are much greater than those of external forces maintaining the vibration. In other words, the external forces are negligible compared to the internal ones. This condition also applies during a natural vibration, since in that case the external forces are zero. Hence the variation of amplitude of vibration from point to point in the system in a condition of resonance is indicated by the normal elastic curve for the type of natural vibration corresponding to the resonance condition.

Just as the application of damping influences has only a very small effect on the frequency of free vibration, so is its effect on the *shape* of the normal elastic curve negligible.

When analysing resonant vibrations in a system it is often desirable to know the relative amplitudes of vibration at different points. These may be determined, without serious loss of accuracy, from the normal elastic curve applying to free undamped vibrations. Damping is taken into account only to determine the absolute amplitude of vibration at any point. Amplitudes at other points are derived from this by use of the normal elastic curve.

In a system composed of masses joined by massless elastic elements, the normal elastic curve consists of straight lines joining the points representing the amplitudes of displacement of the masses.

If the masses of the elastic elements are taken into account, the straight lines become curved; but in many practical cases this effect is negligible, because the elastic elements are extremely light compared with the masses.

1. *Single-frequency System.* The system shown in Fig. 2 is actually a two-mass system, because the upper end of the spring is attached to a support which is assumed not to move under the influence of varying tension in the spring and is therefore equivalent to an infinite mass. The system has a single node situated at the upper end of the spring and a single antinode situated at the lower end.

The system has a single elastic element (the spring), and has therefore only one natural frequency. This statement neglects the mass of the spring itself, as being small compared with that of the suspended mass.

In the simple type of natural vibration already considered the amplitude of vibration varies uniformly along the length of the spring from zero at the upper end to a maximum (defined by the total vibration energy) at the lower end. The normal elastic curve is thus a straight line.

2. *Two-frequency System.* If to the mass shown in Fig. 2 there be added another spring and mass not necessarily of the same dimensions as the original ones, a system results which has two elastic elements (the two springs) and has therefore two natural frequencies corresponding to two different modes of vibration. This new system and its normal elastic curves are represented in Fig. 7.

It will be noted that the higher natural frequency corresponds to a vibration form in which there are two nodes, one in each elastic element. The lower frequency corresponds to the single-node vibration form.

If a pulsating force is applied at any point in the system (excepting the upper end, where infinite mass prevents any measurable response), the system is placed in a state of vibration. If such pulsating force contains a component which is a simple harmonic function of time and whose frequency is equal to one of the natural frequencies of the system, resonance occurs, with high vibration amplitudes having relative values defined by the normal elastic curve for the natural frequency with which resonance has been established.

It is important to note that resonance arises from coincidence between the frequency of *any* single harmonic component of the external force, and *either* natural frequency of the system. If the external force happens to be applied at a point which is a node in the resonant vibration form, excessive building-up of vibration does not occur, and from a practical point of view the condition would hardly be regarded as that of resonance. The technical aspect would include it as a special case of resonance in which the external force is applied at a point where its influence on the system is neither positive nor negative.

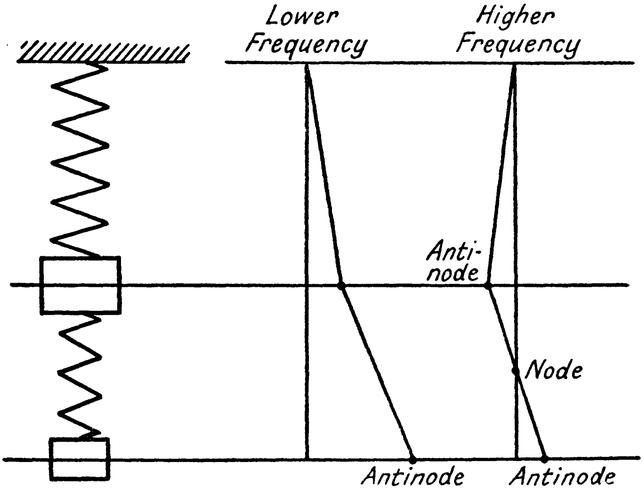


FIG. 7. NORMAL ELASTIC CURVES FOR VIBRATING SYSTEM WITH TWO ELASTIC ELEMENTS

If a number of external pulsating forces are applied at different points in the system their effects are additive, i.e. the resultant amplitude of displacement or of force at any point in the system is the sum of the amplitudes set up there by the different forces taken separately, with appropriate allowance for difference in phase.

If any harmonic component of force is in resonance with the system, the magnification is usually so great that the effect of any other approximately equal harmonic component not in resonance is relatively negligible.

Principle of Superposition. It is possible for a multi-mass system to have a free vibration which is not simple harmonic. This occurs when two or more modes of vibration are taking place simultaneously. The displacement of any point in the system at any instant is the sum of the displacements corresponding to the different modes of vibration. Since the natural frequencies are not, in general, exact multiples of one another, the motion of any point in the system may never exactly repeat itself.

The principle of superposition states that simple harmonic vibrations can occur simultaneously without interfering with each other.

In practice free vibrations are damped by various influences, and those of higher frequency suffer the more rapid loss in amplitude. Consequently a "mixed" vibration, started by some external disturbance, usually resolves itself quickly into a natural vibration of the lowest frequency, and this in turn is gradually extinguished by damping.

Types of Elastic Vibration. Nearly every elastic system is capable of vibrating in more than one manner, and this is illustrated in Fig. 8, which represents two masses in the form of disks connected by an elastic element in the form of a shaft.

The possible types of vibration of the system are—

1. Angular displacement of the disks about their common axis, producing torsional shear stresses in the shaft.

2. Angular displacement of the disks about parallel diameters, producing bending stresses in the shaft.

3. Axial displacement of the disks, producing direct stresses in the shaft.

Vibrations of all these types could occur simultaneously.

In practice it usually happens that only one type of

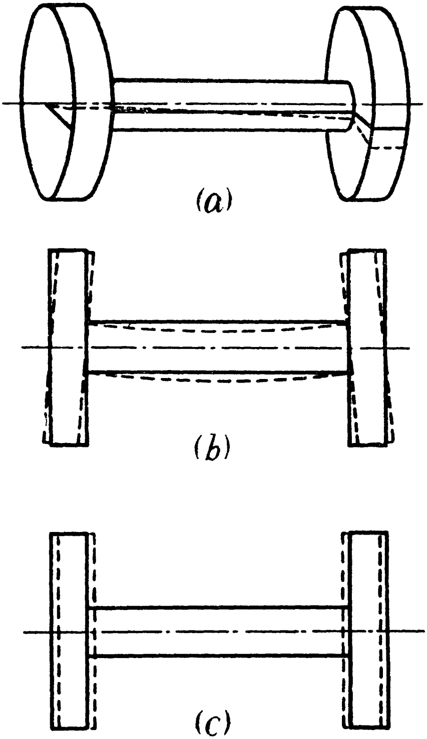


FIG. 8. MODES OF VIBRATION

vibration is likely to be excited, and the others need not be considered. Thus, if the system shown in Fig. 8 were subjected to vibratory torque, the torsional vibration (*a*) would tend to be produced. If in addition a gear were mounted near the mid-point of the shaft, vibratory tooth loads would tend to induce the flexural vibration (*b*). The longitudinal vibration (*c*) is most unlikely to need consideration in practice.

As an illustration of the application of the method described under "Free Vibration," the natural frequencies in the three modes of vibration may be considered in a particular example of the system shown in Fig. 8.

Let Diameter of each disk = 40 in.
,, Thickness of each disk = 5 in.
,, Weight of each disk = 1 780 lb.
,, Length of shaft = 60 in.
,, Diameter of shaft = 7 in.

Material of shaft is steel ($E = 30 \times 10^6$ lb./sq. in.).

1. *Axial Vibration.* Since the arrangement is symmetrical, each disk has the same amplitude of vibration, say λ. The maximum extension of the shaft is 2λ, and therefore—

$$\begin{aligned}\text{Max. strain energy} &= \tfrac{1}{2}\text{ tension} \times \text{extension}\\ &= \tfrac{1}{2}\left(\frac{2\lambda}{60} \times E \times \frac{\pi}{4} \times 7^2\right)2\lambda\\ &= 38{\cdot}5 \times 10^6\,\lambda^2\end{aligned}$$

Let the natural frequency be N cycles/min. Then (see "General Formulae") the maximum vibration energy of the two disks is equal to—

$$2 \times \frac{W\lambda^2 . N^2}{70\,400} = \frac{2 \times 1\,780\lambda^2 . N^2}{70\,400}$$

Equating this to the maximum strain energy—

$$N^2 = \frac{70\,400 \times 38{\cdot}5 \times 10^6}{2 \times 1\,780}$$

or $$N = 27\,600 \text{ cycles/min.}$$

2. *Flexural Vibration.* In this case let the amplitude of angular vibration of each disk be λ. The bending

moment is constant over the length of the shaft, which thus becomes a circular arc of radius $60/2\lambda$. Hence—

$$\text{Bending moment} = \frac{E}{R} \cdot \frac{\pi}{64} \times d^4$$

$$= \frac{30 \times 10^6 \times 2\lambda}{60} \cdot \frac{\pi}{64} \times 7^4$$

$$= 117{\cdot}8 \times 10^6\lambda$$

Strain energy $= \frac{1}{2}$ (B.M.) × relative angular displacement of ends

$$= \tfrac{1}{2}(117{\cdot}8 \times 10^6\lambda)\, 2\lambda$$

$$= 117{\cdot}8 \times 10^6 \lambda^2$$

Moment of inertia of each disk about diameter $= 1\,780\left(\frac{5^2}{12} + \frac{40^2}{16}\right)$

$$= 182\,000 \text{ lb. in.}^2$$

Maximum vibration energy of 2 disks at N cycles/min. $= 2 \times \dfrac{182\,000\,\lambda^2 N^2}{70\,400}$

Equating this to the maximum strain energy—

$$N^2 = \frac{70\,400 \times 117{\cdot}8 \times 10^6}{2 \times 182\,000}$$

or $N = 4\,770$ cycles/min.

3. *Torsional Vibration.* Let the amplitude of angular vibration of each disk be λ. The twisting moment is constant over the length of the shaft in which—

$$\text{Torque} = \text{twist} \times \text{stiffness}$$

$$= 2\lambda \times \frac{1{\cdot}16 \times 10^6}{60/7^4}$$

$$= 93 \times 10^6\lambda$$

$$\text{Strain energy} = \tfrac{1}{2}\,\text{torque} \times \text{twist}$$

$$= \tfrac{1}{2}(93 \times 10^6\lambda)\, 2\lambda$$

$$= 93 \times 10^6 \lambda^2$$

$$\begin{array}{ll}\text{Moment of inertia of each} & = 1\,780 \times 40^2/8 \\ \text{disk about axis} & 356\,000 \text{ lb. in.}^2\end{array}$$

$$\begin{array}{ll}\text{Maximum vibration energy of} & \\ \text{2 disks at } N \text{ cycles/min.} & = 2 \times \dfrac{356\,000\,\lambda^2 N^2}{70\,400}\end{array}$$

Equating this to the maximum strain energy—

$$N^2 = \frac{70\,400 \times 93 \times 10^6}{2 \times 356\,000}$$

or $$N = 3\,030 \text{ cycles/min.}$$

CHAPTER III

VIBRATION-EXCITING INFLUENCES

IN the foregoing consideration of forced vibrations, the assumption has been made that the external exciting influence is of simple harmonic character, because this makes it easier to analyse the conditions mathematically.

In practice, exciting forces are frequently of this character. If, for example, the centre of gravity of a rotating part does not lie on the axis of rotation the part is said to be "unbalanced," and each bearing is subjected to a force of constant magnitude for any speed but of continuously changing direction. The component of such a force in any selected direction is a force varying with time in the simple harmonic manner.

In general, however, disturbing influences of less simple character may have to be considered. For example, the force applied to each crank of an internal combustion engine follows a regular cycle, but the variation of its magnitude during a cycle is not simple harmonic. This is sufficiently indicated by its very sudden rise from a low value to a high one during the brief period of the explosion. Similarly, the torque exerted on the crank is periodic but not simple harmonic.

Harmonic Analysis. It has previously been mentioned that in general any quantity whose value completes, say, 10 cycles/sec. is equivalent to the sum of a number of quantities varying in simple harmonic manner and completing 10, 20, 30, etc., cycles/sec. The process of determining the amplitudes and phase relationship of such quantities is called *harmonic analysis*. Although the rigorous proof of the possibility of such analysis is mathematical, the process itself is purely arithmetical.

The simple harmonic quantities into which a periodically varying quantity may be divided are called *harmonic components*, or simply *harmonics*. The one which has the same frequency as the original quantity is called the *fundamental harmonic*, the one of twice that frequency is the *second harmonic*, and so on. The *order* of a harmonic is the ratio of its frequency to that of the fundamental harmonic.

There can be no harmonic of lower frequency than the original quantity, and by the original definition harmonic orders are confined to whole numbers; an exception occurs, however, in connexion with vibratory forces and torques associated with four-stroke internal combustion engines. There it is conventional to regard the speed of the crankshaft as the fundamental frequency, although the cycle of events in each cylinder has only half that frequency. Consequently harmonic frequencies in that case are multiples of half the crankshaft speed, and, taking this as the fundamental, harmonic orders are $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, etc.

In nearly all practical cases the amplitudes of harmonic components decrease as the order rises beyond 3 or 4. The amplitudes of harmonics of higher than about the twelfth order are usually negligible compared with those of the second or third order. This makes it possible to secure a sufficiently high degree of accuracy from methods of harmonic analysis which are comparatively simple to follow, although inevitably somewhat toilsome if harmonic components of high order are under consideration.

It will be seen that the smaller the amplitudes of the higher harmonic components, the less is the likelihood of the creation of serious resonant vibration. It is for this reason that in generating alternating electric current the aim is to approximate as closely as possible to a "pure sine wave," i.e. to avoid if possible the production of any harmonic components beyond the first.

Thompson Method of Harmonic Analysis. There are many methods of determining the amplitudes of harmonic components of a quantity which varies over a regular cycle. Of these, the one which best combines simplicity and general utility is that due to Professor Sylvanus P. Thompson.

Let the frequency of the quantity be N, and let $n = 2\pi N$. Then the value of the quantity may be represented by—

$$\begin{array}{ll} A_0 \quad . \quad . \quad . \quad . & \text{(mean value)} \\ + A_1 \sin nt + B_1 \cos nt \quad . & \text{(first harmonic)} \\ + A_2 \sin 2nt + B_2 \cos 2nt \quad . & \text{(second harmonic)} \\ + \text{etc.} & \end{array}$$

where A_0, A_1, B_1, A_2, B_2, etc., are constants to be determined from the known variation of the quantity. In general, this series extends to infinity, but the terms beyond the tenth order are usually of negligible amplitude.

The use of a sine term and a cosine term for each frequency is a convenient way of accounting for phase difference.

Any harmonic component (say the rth) is represented by $A_r \sin rnt + B_r \cos rnt$, and this may be written as $\sqrt{(A_r^2 + B_r^2)} \sin(rnt + \varepsilon)$, where $\tan \varepsilon = B_r/A_r$. The amplitude of the rth harmonic is consequently $\sqrt{(A_r^2 + B_r^2)}$, and usually the phase relation between different harmonics is not important. In such a case the term $\sin(rnt + \varepsilon)$ need not be considered.

To determine the amplitude of the rth harmonic it is necessary to know the value of the varying quantity at $4r$ equidistant instants in the cycle. If a number of harmonic components are required, it is convenient to represent the variation of the quantity under consideration by a curve, from which the instantaneous values may be read.

Let the whole cycle be divided into 360 equal parts or "degrees." Let the ordinate (the value of the quantity) at any point distant, say, 6° from the start, be denoted by (6). Then—

$$B_{10} = \frac{1}{20}\begin{bmatrix} +(0) & +(36) & -(18) & -(54) \\ +(72) & +(108) & -(90) & -(126) \\ +(144) & +(180) & -(162) & -(198) \\ +(216) & +(252) & -(234) & -(270) \\ +(288) & +(324) & -(306) & -(342) \end{bmatrix}$$

$$A_{10} = \frac{1}{20}\begin{bmatrix} +(9) & +(45) & -(27) & -(63) \\ +(81) & +(117) & -(99) & -(135) \\ +(153) & +(189) & -(171) & -(207) \\ +(225) & +(261) & -(243) & -(279) \\ +(297) & +(333) & -(315) & -(351) \end{bmatrix}$$

The forms of these expressions show how any coefficients A_r, B_r are determined. To find B_r we add together ordinates, starting at 0 and situated at intervals $360/r$. From this sum we subtract the sum of the corresponding series of ordinates starting at $180/r$. The result is divided by $2r$.

To find A_r the process is identical, but the starting-point for the first series of ordinates is $90/r$ instead of 0, and for the second series $270/r$ instead of $180/r$.

These expressions are not exact, but provided that harmonic components of higher order than the tenth are negligible, the rules are sufficiently accurate down to $r = 4$. Below that the following expressions are to be used—

$$B_3 = \tfrac{1}{6}\,[(0) + (120) + (240) - (60) - (180) - (300)] - B_9$$
$$A_3 = \tfrac{1}{6}\,[(30) + (150) + (270) - (90) - (210) - (330)] + A_9$$
$$B_2 = \tfrac{1}{4}\,[(0) + (180) - (90) - (270)] - B_6 - B_{10}$$
$$A_2 = \tfrac{1}{4}\,[(45) + (225) - (135) - (315)] + A_6 - A_{10}$$
$$B_1 = \tfrac{1}{2}\,[(0) - (180)] - B_3 - B_5 - B_7 - B_9$$
$$A_1 = \tfrac{1}{2}\,[(90) - (270)] + A_3 - A_5 + A_7 - A_9$$
$$A_0 = (0) - (B_1 + B_2 + \text{etc.} \;.\;.\;.\; + B_{10})$$

An alternative method may be used to determine A_0, because it is the mean of any large number of ordinates equidistantly spaced over the cycle.

It will be noticed that before any of the first three harmonics can be evaluated by this method certain higher harmonic amplitudes must be determined.

Example of Harmonic Analysis. Fig. 9 represents the variation of a quantity which continually repeats itself in a cycle represented by the length AB. The fundamental frequency is the reciprocal of AB, i.e. 1/0·85 = 1·178 cycles/sec., or 70·6 cycles/min.

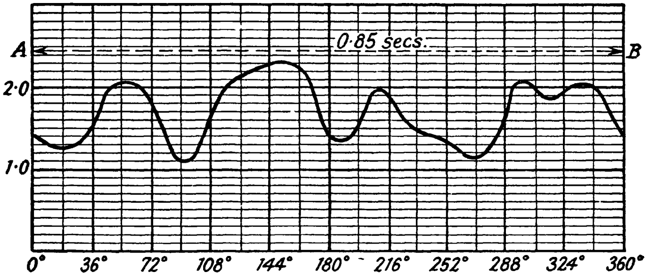


FIG. 9. EXAMPLE FOR HARMONIC ANALYSIS

Let it be required to determine the amplitude of the fifth harmonic component of this varying quantity. The length AB is divided into equal parts, of number equal to four times the order of the required harmonic component. In this case the number is 20, and if the whole cycle is equivalent to 360°, each subdivision is 18°. The heights of the ordinates at these points are measured to be—

(0)	1·38	(90)	1·10	(180)	1·37	(270)	1·14
(18°)	1·26	(108)	1·50	(198)	1·50	(288)	1·65
(36°)	1·52	(126)	2·13	(216)	1·90	(306)	1·95
(54°)	2·02	(144)	2·28	(234)	1·48	(324)	1·90
(72°)	1·82	(162)	2·25	(252)	1·32	(342)	2·00

$$\text{Then } B_5 = \tfrac{1}{10}\begin{bmatrix} (0) + (72) + (144) + (216) + (288) \\ - (36) - (108) - (180) - (252) - (324) \end{bmatrix}$$

$$= \tfrac{1}{10}\begin{bmatrix} 1{\cdot}38 + 1{\cdot}82 + 2{\cdot}28 + 1{\cdot}90 + 1{\cdot}65 \\ - 1{\cdot}52 - 1{\cdot}50 - 1{\cdot}37 - 1{\cdot}32 - 1{\cdot}90 \end{bmatrix}$$

$$= 0{\cdot}142$$

$$A_5 = \tfrac{1}{10}\begin{bmatrix} (18) + (90) + (162) + (234) + (306) \\ - (54) - (126) - (198) - (270) - (342) \end{bmatrix}$$

$$= \tfrac{1}{10}\begin{bmatrix} 1{\cdot}26 + 1{\cdot}10 + 2{\cdot}25 + 1{\cdot}48 + 1{\cdot}95 \\ - 2{\cdot}02 - 2{\cdot}13 - 1{\cdot}50 - 1{\cdot}14 - 2{\cdot}00 \end{bmatrix}$$

$$= - 0{\cdot}075$$

The amplitude of the fifth harmonic component is then—

$$\sqrt{(0{\cdot}142^2 + (-0{\cdot}075)^2)} = 0{\cdot}161$$

Its frequency is $5 \times 1{\cdot}178 = 5{\cdot}89$ cycles/sec., or 353 cycles/min. Its "frequency constant" is $2\pi \times 5{\cdot}89 = 37$. Its phase angle α is given by $\tan \alpha = B/A = 0{\cdot}142/{-0{\cdot}075} = -1{\cdot}89$, whence $\alpha = -62° \, 7' = -1{\cdot}084$ radians.

It is completely expressed as a function of time in seconds measured from the instant A by the expression

$$\sqrt{(A^2 + B^2)} \sin (\omega t + \alpha) = 0{\cdot}161 \sin (37t - 1{\cdot}084)$$

Critical Speeds. The most troublesome form of vibration usually encountered in practice is that which occurs in the condition of resonance. It has already been mentioned that the magnification in that condition may be of the order of 600, so that a vibration-exciting force too small to be serious in normal circumstances may give rise to objectionable vibration when its frequency is equal to a natural frequency of the vibrating system.

Now, an assembly of machinery usually has several different modes of vibration and has as many different natural frequencies. Also, the vibration-exciting forces set up within it may not be of simple harmonic character, or, in other words, they generally contain harmonic components of higher frequencies than the fundamental one. A condition of resonance occurs if the frequency of any of these harmonic components coincides with any one of the natural frequencies. If the machine's speed is variable (and with it the fundamental frequencies of exciting forces), such coincidence may be unavoidable. Fortunately, however, not all resonance conditions produce dangerously high stresses, for the harmonic components may be of small magnitude, or the damping effect may be considerable. Furthermore, equal exciting forces may be applied at such points in the vibrating system that their effects tend to cancel out.

A running speed which produces resonance is called a *critical speed*. In identifying a critical speed it is necessary to state the number of nodes in the type of vibration with which resonance is established at that speed, and also the order of the harmonic component which produces the resonant vibration. For example, if the two-node vibration has a frequency of 10 000 cycles/min., a running speed of 1 250 r.p.m. will cause the eighth order harmonic component to be in resonance with that type of vibration, since $1\,250 \times 8 = 10\,000$. The speed of 1 250 r.p.m. is therefore the two-node, eighth order critical speed.

This illustrates the method of determining critical speeds. It consists simply of dividing the natural frequencies by whole numbers (or in the case of four-stroke internal combustion engines, halves of whole numbers, i.e. $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, etc.). The process of determining which speeds out of this range are really critical, in the sense of inducing dangerously severe vibration, is more difficult.

Major and Minor Critical Speeds. Where a vibration is maintained by harmonic-exciting influences of equal frequency, the phase relationship between those influences has to be considered in determining the total effect.

In a multi-cylinder engine, the phase difference between the harmonic components of any one order produced in different cylinders is dependent on the difference between the crank angles for the cylinders and upon the order of the harmonic component. If the phase difference is 360° or any integral multiple of 360°, it is equivalent to zero difference and the harmonic components concerned directly assist each other.

A speed which causes the vibrating system to be in resonance with harmonic components which are in phase with each other is called a *major critical speed.* All other critical speeds are classed as *minor critical speeds.*

In an internal combustion engine a major critical speed is one whose harmonic order is equal to, or is an integral multiple of, the number of firing strokes made by the engine in one revolution of the crankshaft. At such a speed the resonant harmonic components of the torques at all cranks are in phase with each other, and their effect in building up resonant torsional vibrations is accordingly comparatively great.

CHAPTER IV

TORSIONAL VIBRATION

IT is in connexion with internal combustion engines that serious critical speeds are most numerous, and the type of vibration concerned is that associated with twisting of the crankshaft and of any other shafts connected to it. The vibration-exciting elements are the harmonic components of the torques exerted on the cranks. The vibrating system consists of the various parts of the shafts which are the elastic elements, and the various massive rotating parts whose moments of inertia correspond to the masses of the suspended weights in the system shown in Fig. 7.

The reason why the internal combustion engine is mentioned as being prone to torsional vibration in distinction from, say, the steam engine or reciprocating pump is that the torque curve of the first-mentioned type of machine is more markedly irregular, and this means that the harmonic components of high order are of appreciable magnitude. This remark applies with particular force to the compression-ignition engine.

The nature of torsional vibration as manifested at any particular point in an engine is that of an angular vibration on each side of a uniformly rotating angular position. Difference between such vibrations at two points on a shaft implies that the angle of twist is varying, and therefore that there is a corresponding torque variation in the shaft. Torsional fatigue failures of crankshafts under the influence of high stresses of this kind produced by resonance first called attention to torsional vibration as a phenomenon of practical importance.

In most engines a critical speed for torsional vibration tends to show itself by noisy running noticeable by an experienced observer. The rapidly varying circumferential velocities of the crankpins in the condition of resonant torsional vibration produce impacts at the big ends of the connecting rods with corresponding reactions at the main bearings. The severity of such impacts is naturally dependent on the bearing clearances, but even in favourable conditions the effect is shown by a "roughness" in the running of the engine.

It must not be supposed that troublesome torsional vibration is confined to engine drives. It is possible for appreciable resonant vibration to arise out of torque variation which would otherwise be negligible. For this reason it is desirable to calculate the natural frequencies for torsional vibrations of any proposed system incorporating heavy rotating parts and comparatively long shafts.

Example of Calculation of Natural Frequency of Two-mass Torsionally Vibrating System. Fig. 10 represents a marine propeller drive, each of the six rectangles nearest the left-hand side corresponding to the equivalent moment of inertia of the moving parts associated with one cylinder. The seventh rectangle corresponds to the flywheel, and that at the right-hand end corresponds to the propeller. Each moment of inertia is the product of weight in pounds, and the square of radius of gyration in inches.

The torsional stiffness of the crankshaft between adjacent throws has been determined from the formula for twist given in Table I. The equivalent lengths of shaft of 11 in. diameter are determined by—

$$l = \frac{1{\cdot}16 \times 10^6 \times 11^4}{(\text{Stiffness in lb. in./radian})}$$

As the stiffness of the shafting within the engine itself is very much greater than that of the long shaft connecting flywheel to propeller, the angle of twist in the long shaft in the single-node type of vibration is much greater than that in the crankshaft. Consequently the flywheel and cranks have almost identical amplitudes of vibration, and the sum of the seven moments of inertia may be considered as that of a single mass situated at their mean centre.

The distance of this imaginary mass from the flywheel

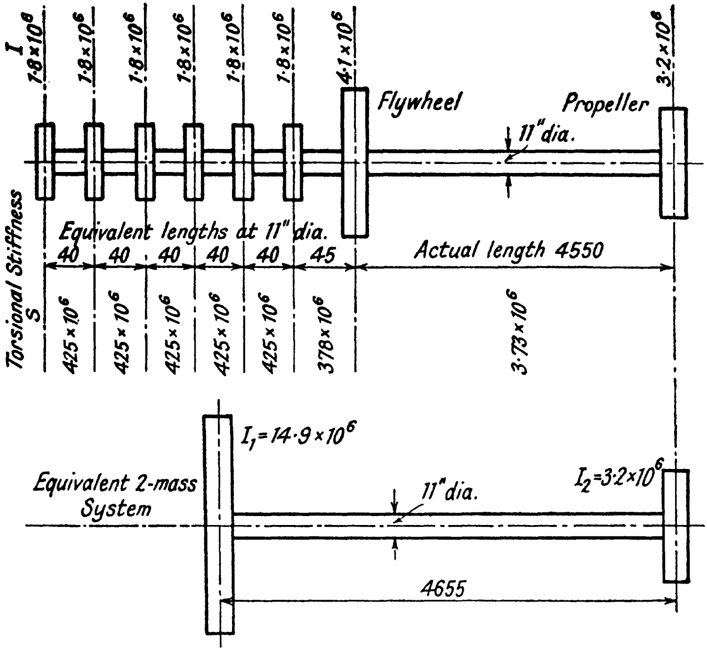


FIG. 10. MARINE ENGINE AND PROPELLER DRIVE

is determined by taking moments about the flywheel thus—

$$1{\cdot}8\times10^6[45+85+125+165+205+245] = x[6\times1{\cdot}8\times10^6+4{\cdot}1\times10^6]$$
$$= 14{\cdot}9\times10^6x$$

whence $x = 105$

Hence the system is reduced to one of two masses of moments of inertia $14{\cdot}9 \times 10^6$ and $3{\cdot}2 \times 10^6$ at opposite ends of a shaft 11 in. in diameter and $105 + 4\,550 = 4\,655$ in. long.

The torsional stiffness of this shaft is determinable from the general formula given alongside the fifth diagram in Table I. Here there is only one diameter to be considered, and therefore—

$$\theta = \frac{1}{C} \cdot \frac{l}{J}$$

or Stiffness $s = \frac{CJ}{l} = \frac{11{\cdot}8 \times 10^6}{l} \times \frac{\pi d^4}{32}$ (for steel shaft, diameter d)

$$= 1{\cdot}16 \times 10^6 \times d^4/l \text{ lb. in./radian}$$

Here $s = 1{\cdot}16 \times 10^6 \times 11^4/4\,655 = 3{\cdot}65 \times 10^6$.

For two-mass system—

Natural frequency

$$= 188 \sqrt{\left[s\left(\frac{1}{I_1} + \frac{1}{I_2}\right)\right]} \text{ cycles/min.}$$

$$= 188 \sqrt{\left[3{\cdot}65 \times 10^6\left(\frac{1}{14{\cdot}9} + \frac{1}{3{\cdot}2}\right)\frac{1}{10^6}\right]}$$

$$= 221 \text{ cycles/min.}$$

Geared Systems. Systems of rotating parts including gears need special consideration when investigating their torsional vibration characteristics. The gears themselves are more adversely affected by torsional vibration than are other parts of the system, because it is necessary in practice to provide backlash in gears, and consequently cyclic reversal of torque causes the backlash to be taken up first in one direction and then in the other. The result is a strong tendency to noisy operation and to rapid wear of the teeth by reason of repeated impacts.

The general procedure in analysing a geared system is to reduce it to an equivalent "ungeared" system, running at the same mean speed as any selected shaft in the original system. If, in constant-speed condition, shafts Nos. 1, 2, 3, etc., run at G_1, G_2, G_3, etc., times the speed of the reference shaft, the "ungeared" equivalent of any torsional stiffness in shaft w is G_w^2 times that stiffness, and the "ungeared" equivalent of any moment of inertia associated with shaft w is also G_w^2 times that moment of inertia.

Fig. 11 gives details of a system of shafts including two pairs of gears and two additional members of appreciable moment of inertia. Except for those parts of the shafts which form the pinions, the masses of the shafts are neglected in the following.

Making allowance for changes in shaft diameter and for keying of hubs on shafts, as described in Note 1 to Table I, the stiffnesses of the shafts are obtained thus—

$$AB: \quad s_1 = \frac{1{\cdot}16 \times 10^6}{\dfrac{2{\cdot}5 + 10 + 0{\cdot}3}{5^4} + \dfrac{7 - 0{\cdot}3 + 0{\cdot}39}{6^4}}$$
$$= 44{\cdot}7 \times 10^6$$

$$CD: \quad s_2 = \frac{1{\cdot}16 \times 10^6}{\dfrac{3{\cdot}0 + 16{\cdot}5 + 0{\cdot}55}{6^4}}$$
$$= 75 \times 10^6$$

$$EF: \quad s_3 = \frac{1{\cdot}16 \times 10^6}{\dfrac{4{\cdot}0 + 8 - 0{\cdot}38}{8^4} + \dfrac{0{\cdot}38 + 15 + 3{\cdot}5}{7^4}}$$
$$= 108{\cdot}8 \times 10^6$$

The equivalent dynamical system may be based on any one of the three shafts. Taking shaft No. 3 as the reference shaft, the stiffnesses and moments of inertia in the equivalent system are derived thus—

Quantity	Actual System	(Velocity Ratio)^2	Equivalent System	
I_1	120 000	$5{\cdot}82^2 = 33{\cdot}9$		4 070 000
s_1	$44{\cdot}7 \times 10^6$	$5{\cdot}82^2 = 33{\cdot}9$		$1\,516 \times 10^6$
I_2	1 270	$5{\cdot}82^2 = 33{\cdot}9$	43 100	90 800
I_3	15 600	$1{\cdot}75^2 = 3{\cdot}06$	47 700	
s_2	75×10^6	$1{\cdot}75^2 = 3{\cdot}06$		$229{\cdot}5 \times 10^6$
I_4	4 840	$1{\cdot}75^2 = 3{\cdot}06$	14 800	60 200
I_5	45 400	1	45 400	
s_3	$108{\cdot}8 \times 10^6$	1		$108{\cdot}8 \times 10^6$
I_6	200 000	1		200 000

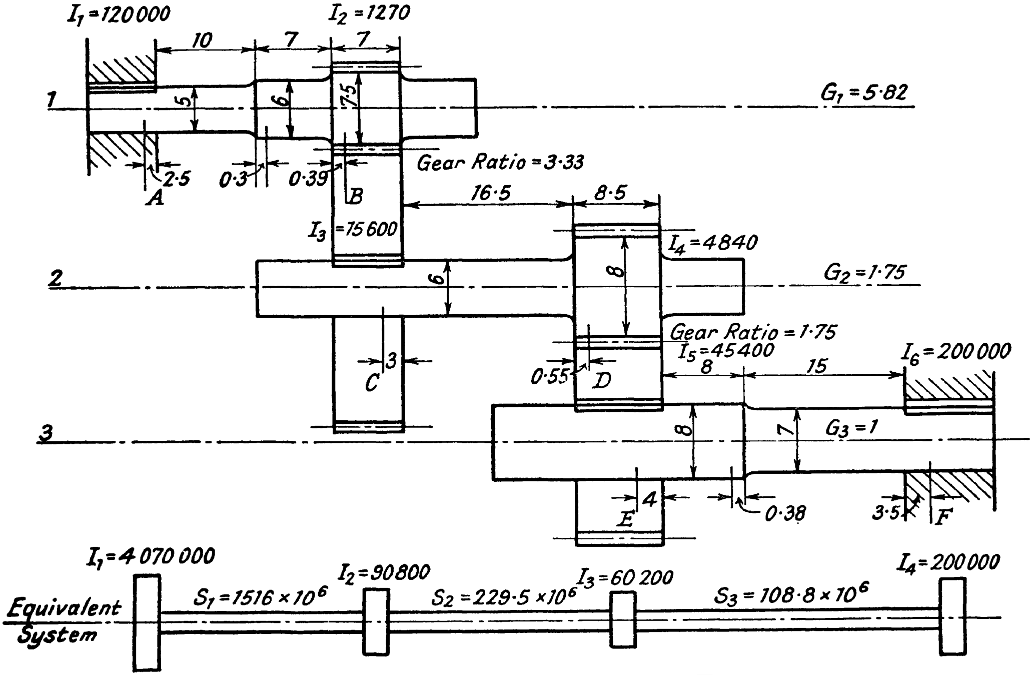


FIG. 11. GEARED SYSTEM AND ITS DYNAMIC EQUIVALENT

The equivalent system is shown in the lower part of Fig. 11. This is a four-mass system, for which the natural frequencies are most conveniently found by the general method already described under Multi-mass Systems and illustrated later. They cannot be determined by any such simple formula as applies to a two-mass system.

Example of Calculation of Natural Frequency of Multi-cylinder Engine. To illustrate the calculation of the lowest natural frequency of torsional vibration of an internal combustion engine, the details given in Fig. 12 may be considered. The dynamical system has the equivalent of seven separate masses, one associated with each cylinder and one with the flywheel.

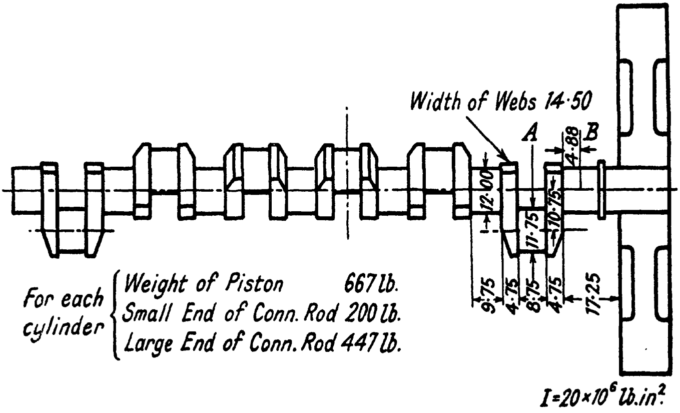


FIG. 12. CRANKSHAFT OF SIX-CYLINDER ENGINE

The moment of inertia of each cylinder-line mass is composed of—

(*a*) The moment of inertia of the crank webs;

(*b*) The moment of inertia of the crankpin;

(*c*) The equivalent moment of inertia of the connecting rod;

(*d*) The equivalent moment of inertia of the piston.

The values for (*a*) and (*b*) may be determined with the aid of formulae contained in Table II, appropriate allowances being made for the lightening of crank webs by chamfering.

If the weight of the connecting rod is W, and its centre of gravity is distant b_1 from the centre-line of the crankpin and b_2 from the centre-line of the gudgeon pin, it is regarded as equivalent to a weight $Wb_2/(b_1+b_2)$ situated at the crankpin (big-end) and a weight $Wb_1/(b_1 + b_2)$ situated at the piston (small end).

The moment of inertia corresponding to the first of these is $\dfrac{Wb_2}{b_1 + b_2} \, . \, r^2$ where r is the crank radius, i.e. half the stroke of the piston. The other quantity is added to the weight of the piston to give the total reciprocating weight.

The equivalent moment of inertia of a reciprocating weight varies with the angular position of the crank in relation to the line of reciprocation. This variation introduces such mathematical complications that it is impracticable to account for it accurately in calculating natural frequencies. The mean kinetic energy of a reciprocating mass is half that of an equal mass which has circular motion of the same amplitude and frequency, and it is found satisfactory for the present purpose to replace a reciprocating weight by a rotating one of half the amount moving on a circle of diameter equal to the range of reciprocation.

In this instance, the total reciprocating weight is $\dfrac{Wb_1}{b_1 + b_2} + W_p$, where W_p is the weight of the piston, and the moment of inertia corresponding to the reciprocating weights is $\frac{1}{2}\left(\dfrac{Wb_1}{b_1 + b_2} + W_p\right)r^2$.

In Fig. 12 the separate weights arising out of the imaginary division of the connecting rod are specified. The total reciprocating weight on each cylinder line is therefore 667 + 200 = 867 lb. The moment of inertia for each cylinder line is—

Crank (calculated)		= 76 000 lb. in.^{2}
Piston and small end (reciprocating)	$\frac{1}{2} \times 867 \times 10{\cdot}75^2 =$	50 000 lb. in.^{2}
Big-end	$447 \times 10{\cdot}75^2 =$	51 500 lb. in.^{2}
Total	I_c	= 177 500 lb. in.^{2}
Flywheel	I_0	= 20×10^6 lb.in.^{2}

The torsionally elastic elements in the system are the lengths of crankshaft between adjacent big-ends, and the length between the sixth big-end and the flywheel. The torsional stiffness of the first of these is equal to that of one crank throw between the centre-line of adjacent main bearings. It is determinable from the formula for angle of twist, given in Table I. This formula is empirical, and others may be considered. (See ref. 36.)

Using Table I, the angle of twist in the length of shaft between adjacent cylinders for unit torque is—

$$\frac{10{\cdot}2}{11{\cdot}8 \times 10^6}\left[\frac{9{\cdot}75 + 1{\cdot}9}{12^4} + \frac{1{\cdot}1 \times 8{\cdot}75}{11{\cdot}75^4} + \frac{1{\cdot}28 \times 10{\cdot}75}{4{\cdot}75 \times 14{\cdot}5^3}\right]$$

$$= \frac{0{\cdot}001735}{10^6}$$

The torsional stiffness of shaft between cylinders is the reciprocal of this, i.e. $s_c = 576 \times 10^6$ lb. in./radian.

Of the length of 17·25 in. between crank No. 6 and the flywheel, the right-hand half *AB* of the crank includes half of 9·75, i.e. 4·88 in.

As the collar is narrow compared with the diameter of the shafts, its torsional stiffening effect is negligible. For the entry of the shaft into the flywheel a length allowance equal to half the diameter is made in accordance with Note 1 to Table I. Hence the effective length of shaft between B and the flywheel is $17{\cdot}25 + (12/2) - 4{\cdot}88 = 18{\cdot}37$ in. at 12 in. diameter, and the angle of twist for unit torque (by Table I) is—

$$\frac{1}{11{\cdot}8 \times 10^6} \cdot \frac{18{\cdot}37}{(\pi/32)12^4} = \frac{0{\cdot}000766}{10^6}$$

Hence the angle of twist for unit torque in the shaft between cylinder No. 6 and the flywheel is the sum of

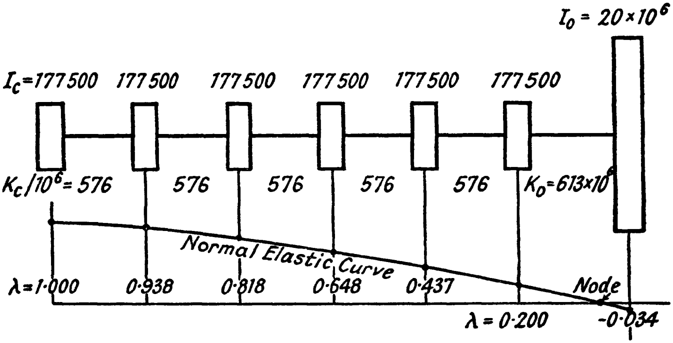


FIG. 13. EQUIVALENT DYNAMICAL SYSTEM FOR SIX-CYLINDER ENGINE

this quantity and half the twist in a complete crank-throw, i.e.—

$$\frac{0{\cdot}000766 + 0{\cdot}5 \times 0{\cdot}001735}{10^6} = \frac{1}{613 \times 10^6}$$

and hence the torsional stiffness of the shaft between cylinder No. 6 and the flywheel is—

$$s_0 = 613 \times 10^6 \text{ lb. in./radian.}$$

The dynamical system is represented by Fig. 13. It is a special type, inasmuch as it is composed of a number of equal rotors (moment of inertia I_c) connected together by equal shafts (stiffness s_c), one end of this assembly being connected by a shaft of different stiffness (s_0) to a larger rotor (moment of inertia I_0).

The number of natural frequencies of torsional vibration of such a system is equal to the number of its shafts, but in practice it is usually only the lowest frequency (single-node vibration) that is important.

For this special type of system—

Lowest natural frequency $= 188\sqrt{(Ds_c/I_c)}$ cycles/min.

where $D = A + BI_c/I_0 + Cs_c/s_0$ (s_0 and s_c are in lb. in./radian; I_0 and I_c are in lb. in.^{2}), and A, B, and C are numerical factors obtained from Table IV (page 102), which covers every number of equal rotors from 4 to 10.

In this case—

$$D = 0{\cdot}0753 + 0{\cdot}294 \times \frac{177\ 500}{20 \times 10^6} - 0{\cdot}0171 \times \frac{576 \times 10^6}{613 \times 10^6}$$

$$= 0{\cdot}0618$$

Hence—

$$\text{Lowest natural frequency} = 188\sqrt{\frac{0{\cdot}0618 \times 576 \times 10^6}{177\ 500}}$$

$$= 2\ 660 \text{ cycles/min.}$$

This corresponds to the single-node vibration form, as is shown later. Higher natural frequencies cannot be determined by any simple formula; consequently the general method described below must be used for them.

Critical speeds for single-node torsional vibration in this engine are obtained by dividing 2 660 by 1, 2, 3, 4, etc., if it is a two-stroke engine or by $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, etc., if it is a four-stroke engine. In the former case, the engine makes six firing strokes per revolution, and the major critical speeds are those whose orders are multiples of 6. In the case of the four-stroke engine, the orders of major critical speeds are multiples of 3.

CRITICAL SPEEDS ASSOCIATED WITH NATURAL FREQUENCY 2 660 CYCLES/MIN.

(M indicates Major Critical Speed)

Two-stroke Engine		Four-stroke Engine	
Order	Critical Speed	Order	Critical Speed
—	—	½	5 320
1	2 660	1	2 660
—	—	1½	1 772
2	1 330	2	1 330
—	—	2½	1 064
3	887	3 *M*	887
—	—	3½	760
4	665	4	665
—	—	4½	592
5	532	5	532
—	—	5½	484
6 *M*	443	6 *M*	443
—	—	6½	409
7	380	7	380
etc.	etc.	etc.	etc.

General Method for Determining Natural Frequencies. In general, there is no simple formula by which to determine the natural frequency of a torsionally vibrating system having a large number of masses. It is usual to estimate it and then to test the accuracy of the estimate by the method described under Multi-mass Systems and detailed below. The test, applied to the lowest natural frequency of the engine drive just discussed, is shown here.

Frequency-constant corresponding to 2 660 cycles/min. $= n = (2\,660/60) \times 2\pi$
$= 278$

In the following, the moments of inertia have to be expressed in a unit equal to 386 lb. in.^{2} in order to maintain consistency with stiffness measured in lb. in./radian. Each moment of inertia is expressed as $i = I/386$.

The test calculation is carried out in tabular form thus—

TEST FOR $n^2 = 278^2 = 77\,300$

i	$\frac{n^2 i}{10^6}$	λ	$\frac{n^2 i \lambda}{10^6}$	$\sum \frac{n^2 i \lambda}{10^6}$	$\frac{10^6}{s}$	$\frac{1}{s}\Sigma n^2 i \lambda$
460	35·6	1	35·6	35·6	0·001735	0·0618
460	35·6	0·9382	33·4	69·0	,,	0·1198
460	35·6	0·8184	29·2	98·2	,,	0·1706
460	35·6	0·6478	23·05	121·25	,,	0·2105
460	35·6	0·4373	15·58	136·83	,,	0·2375
460	35·6	0·1998	7·10	143·93	0·001630	0·234
51 800	4 000	− 0·0342	− 141·5	2·43	—	—

In the first column the moments of inertia are entered. The figures in the second column are those in the first one multiplied by n^2 and (simply to avoid inconveniently large numbers in the body of the table) divided by 10^6. To balance this, the sixth column, which otherwise would contain figures representing the reciprocals of stiffnesses ($1/s$) is a list of the values $10^6/s$ for successive shafts starting from the same end of the system as do the moments of inertia in column 1. (Note that the last line in column 6 is blank.)

The amplitudes λ of angular displacement appear in the third column. The value of λ in the first line is written equal to unity, and the fourth entry in each line is the product of the second and third entries. The fifth entry in each line is the sum of the fourth in that line and the fifth in the preceding line. Each quantity in the last column is the product of the corresponding quantities in the two preceding columns. It is subtracted from the λ in the same line to give the λ for the succeeding line.

This procedure is a simplified routine for the method described previously. Each quantity in column 4 represents the amplitude of torque on that rotor. Each quantity in column 5 represents the total torque on that rotor and all preceding ones. Each quantity in column 7 represents the angle of twist produced in the corresponding shaft by the total torque on all rotors preceding it. Consequently subtraction of that quantity from λ in the same line gives the amplitude of angular displacement at the next rotor.

When the last rotor is reached there is no further shaft to take the total torque on all the rotors. Consequently that total, i.e. the last entry in column 5, must be zero if the selected value of n corresponds to a natural frequency.

In the particular example shown, the last entry is 2·43, which is so small compared with other quantities in column 5 that $n^2 = 77\ 300$ may be accepted as closely corresponding to a natural frequency. It will be found that a small increase in n^2 causes the final value to change from 2·43 to a negative quantity, showing that the correct value of n^2 is very slightly greater than 77 300.

The values in the third column are the relative amplitudes of angular vibration at the corresponding rotors. The fact that there is a change of sign between 0·1998 and − 0·0342 indicates that there is a node between the sixth and seventh rotors. As this is the only such change, the vibration is the single-node form, and 2 660 c.p.m. is therefore the lowest natural frequency. The values of λ are plotted on Fig. 13, and the points thus obtained mark out the normal elastic curve for the single-node vibration form.

Although this illustration refers to a system containing a number of equal masses, the method applies equally well to any general system of shafts and rotating masses.

Torsional Stresses Produced at Critical Speeds. The severity of torsional vibration at any particular critical speed depends on—

(*a*) The amplitude of the harmonic torque components exciting the resonant vibration;

(*b*) The phase differences between the harmonic torque components;

(*c*) The positions of the points of application of the exciting torques relatively to the node or nodes in the resonant vibration form.

(*d*) The magnitudes of the damping effects restraining the building-up of resonant vibration.

Referring to (*a*), Table V (page 102) shows the approximate values of harmonic components of the torque exerted on the crank of a compression-ignition engine as multiples of the mean torque exerted on the crank.

The phase differences between harmonic components of the same order are equal to the angles between the cranks concerned multiplied by the order of the harmonic component.

In an internal combustion engine direct-coupled to the driven machine, the node in the single-node vibration form is usually close to the flywheel, the non-driving end of the crankshaft being an antinode. Vibratory torques exerted near that end have therefore a comparatively large influence in producing resonant torsional vibrations.

The damping effects cannot be accurately calculated for *every* particular case, but so many have been investigated in the past that a reasonably close estimate may be made of them for any new design. The more important ones are—

(*a*) Mechanical hysteresis in shafts and in other parts subjected to vibratory stresses produced by torsional vibration;

(*b*) Loss of energy produced by variation of load on oil films in bearings;

(*c*) Variation of resisting torque with speed in certain types of driven machine, e.g. electrical generators, marine propellers.

If the installation is required to operate at only one speed, it is usually possible so to arrange the torsional stiffnesses of the shafts and the moments of inertia of the moving parts that the running speed is well removed from any critical speed. In starting up or shutting down it may be necessary to pass through one or more critical speeds, but this is rarely serious, firstly because the full amplitude of vibration cannot be built up by momentary resonance, and secondly because even the maximum vibration stresses possible with sustained running at the critical speed are not usually high enough to produce failure immediately, but only by fatigue. For this reason calculation of stresses at critical speeds is not usually necessary in the case of a constant-speed drive.

If the installation is required to operate over any considerable range of speed, avoidance of critical speeds may be impossible. In that case an attempt should be made to keep the running speed range free from major critical speeds.

Generally speaking, the lower the order of a harmonic exciting torque, the greater is its amplitude. For this reason it is desirable in general to use stiff shafts so that the natural frequencies of torsional vibration of the assembly may be high. Then the harmonic components which may produce critical speeds within a certain range are of high order, and are consequently comparatively weak.

Fig. 14 shows a typical distribution of torsional stresses at critical speeds for a six-cylinder oil engine coupled to an electric alternator.

Although it may be found impossible in some particular cases to exclude critical speeds from the running range, the stresses at such speeds may not be seriously high. There may, however, be other reasons why the critical speeds are objectionable. For example, camshaft driving gears situated at the end of a multi-cylinder internal combustion engine opposite to the flywheel suffer a relatively large amplitude of angular vibration at a single-node critical speed, and the noise which they produce in that condition (owing to reversal of backlash between the teeth) may be unacceptable. Again, the fluctuations of angular velocity of the armature of the electrical machine in an engine-driven lighting set may be sufficiently severe at a critical speed to produce flicker in the lights.

Method of Calculating Amplitude of Resonant Vibration. The full details of calculation of stresses at critical speeds cannot be given here, but the general principle may be indicated. The amplitudes of displacement in the resonant condition are expressed as multiples of the amplitude λ_0 at a selected point O. The vibration energy input per cycle is then determined in terms of the amplitude of the exciting forces or torques.

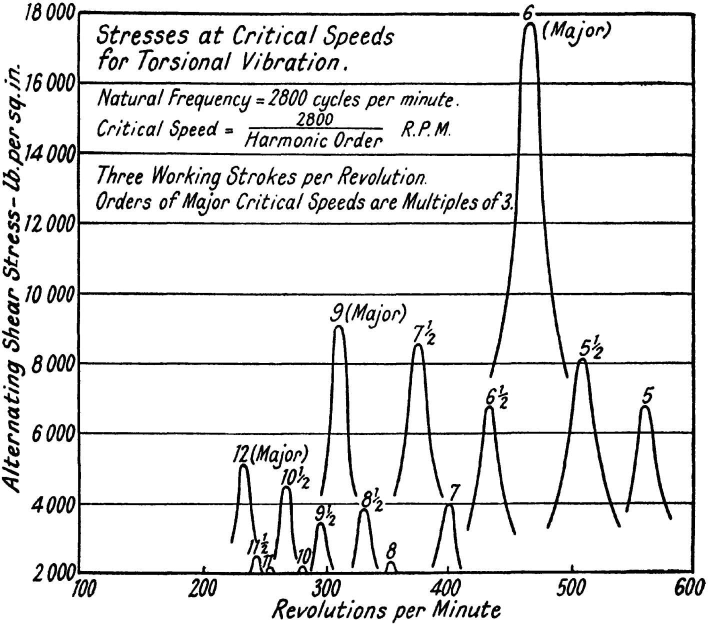


FIG. 14. STRESSES DUE TO TORSIONAL VIBRATION AT VARIOUS SPEEDS IN SIX-CYLINDER FOUR-STROKE OIL ENGINE

If forces $F_1 \sin(pt + \alpha_1)$, $F_2 \sin(pt + \alpha_2)$, etc., act at points where the amplitudes are multiples U_1, U_2, etc., of the amplitude at O, the energy input per cycle by all forces in the resonance condition is—

$$Z = \pi\, \lambda_o \surd^4[(\Sigma UF \cos \alpha)^2 + (\Sigma UF \sin \alpha)^2] \text{ in. lb. } . \quad (7)$$

In the resonance condition this quantity is equal to the total energy dissipated per cycle by all damping effects. (In torsional vibration the F's are torques and the λ's are angles.)

By using the normal elastic curve, the load in every elastic member can be expressed in terms of λ_o, and data on mechanical hysteresis may then be used to express the energy loss per cycle in terms of λ_o. It is proportional to λ_o raised to some power higher than 2.

In the case of torsional vibration there may be a resisting torque $L\omega^r$ (where L and r are constants and ω is the angular velocity) applied at a point (e.g. pump impeller) where the amplitude of vibration is P times that at O. If the frequency of the resonant vibration is N cycles/min., the energy dissipated per cycle at this point is—

$$(\pi^2/30)\, NrL\omega^{r-1}P^2\, \lambda_o^2$$

By equating the total energy loss to the energy input, the value of λ_o is obtained for the resonance condition. Reference to the normal elastic curve then gives the amplitudes of displacement at all points, and from these the loads and stresses in all parts of the system can be calculated.

Amplitude of Resonant Torsional Vibration in Engine. In applying equation (7) to torsional vibration in a multi-cylinder engine, the angles α_1, α_2, etc., are simply related to the crank angles. It is permissible to write $\alpha_1 = 0$, since time may be measured from any instant. Then the angle α_W applying to any other cylinder is equal to Y times the angle between crank W and crank No. 1, where Y is the order of the resonant harmonic. In dealing with four-stroke engines, allowance must be made for the fact that one cycle occupies two revolutions, the range of crank angle thus being 720°.

If the order of the resonant harmonic is a multiple of the number of firing strokes made by the engine in each revolution (as occurs at a major critical speed), it is found that each α is a multiple of 360°. Consequently equation (7) becomes simplified to—

$$Z = \pi F \lambda_o \Sigma U$$

since the F's for all cylinders are the same.

In a shaft system such as that shown in Fig. 12 (page 41) damping is almost confined to hysteresis and other internal friction (corresponding to (*a*) and (*b*) above) in the moving parts of the engine. A dynamically similar system is that in which such an engine is rigidly coupled at the flywheel end to the rotor of an electrical generator. Such a driven machine is normally capable of exerting damping effect as described under (*c*) above, but in this particular type of system the amplitude at the flywheel is so small compared with those at the cylinders that the damping influence of the generator is negligible.

It is extremely difficult to evaluate the various damping influences, but observations taken on many installations of this type show that it is safe to assume a magnification factor at resonance of about 75.

Now

$$\text{Vibration energy of system} = \frac{N^2}{70\,400}\Sigma I \lambda^2 = \frac{N^2}{70\,400}\lambda_o^2 \Sigma I U^2$$

and

$$\text{Magnification factor at resonance} = \frac{2\pi \text{ vibration energy}}{\text{energy loss per cycle}}$$

Equating these, and writing Magnification = 75—

$$75\pi\,\lambda_o\sqrt{[(\Sigma UF\cos\alpha)^2 + (\Sigma UF\sin\alpha)^2]} = 2\pi\frac{N^2\lambda_o^2}{70\,400}\Sigma I U^2$$

and

$$\lambda_o = \frac{2{\cdot}64 \times 10^6}{N^2 \Sigma I U^2}\sqrt{[(\Sigma UF\cos\alpha)^2 + (\Sigma UF\sin\alpha)^2]} \qquad (8)$$

where N is in cycles/min.,
the I's are in lb. in.^{2},
the F's are in lb. in.,
and λ_0 is in radians.

The U's are ratios taken from the normal elastic curve, and the summation Σ applies to all the cylinders.

This gives the amplitude of vibration at the selected point O. From this the amplitudes at any other points may be determined from the normal elastic curve by simple proportion, and hence the torque and stress in any shaft may be calculated.

In the example shown in Figs. 12 and 13 (pages 41 and 44) let the point O be crank No. 1, and let the calculated value of λ_0 be 0·009 radian. Then from the normal elastic curve the angle of twist between crank No. 6 and the flywheel is 0·200—(— 0·034) = 0·234 times that at crank No. 1, i.e. 0·00211 radian. Hence—

$$\text{Torque} = \text{stiffness} \times \text{twist} = 613 \times 10^6 \times 0{\cdot}00211$$
$$= 1{\cdot}295 \times 10^6 \text{ lb. in.}$$

$$\begin{array}{l}\text{Amplitude of vibratory} \\ \quad\text{torsional shear stress in} \\ \quad\text{12 in. dia. shaft}\end{array} = \frac{1{\cdot}295 \times 10^6 \times 5{\cdot}1}{12^3}$$
$$= 3\,830 \text{ lb./in.}^2$$

Effect of Firing Order. The energy input at any minor critical speed is dependent on the firing order. An alteration in this may therefore diminish resonant stresses at such a speed, although at the expense of increased stresses at other minor critical speeds.

No change in firing order can affect resonant stresses at a major critical speed.

CHAPTER V

REDUCTION OF VIBRATION STRESSES AT CRITICAL SPEEDS

IF no permissible variation in mass and flexibility in a vibrating system suffices to remove a certain critical speed from the running range and it is feared that the stresses produced at that speed will be dangerous unless provision is made for reducing them, it becomes necessary to—

(*a*) introduce a "vibration damper" which dissipates vibration energy when resonance is established and thus reduces the amplitude of vibration in that condition; or

(*b*) introduce a "dynamic vibration absorber" which takes most or all of the effect of the vibration-exciting force or torque and thus diminishes its influence on the main part of the vibrating system; or

(*c*) use a "damped vibration absorber," which is a combination of (*a*) and (*b*).

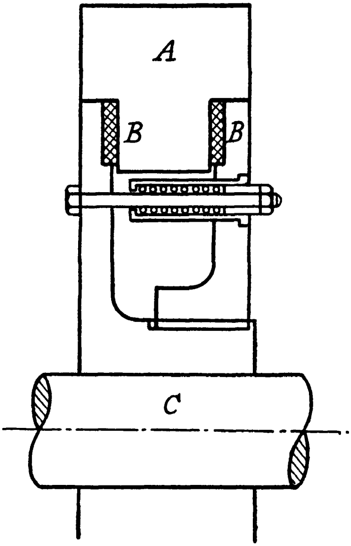


FIG. 15. TORSIONAL VIBRATION DAMPER (LANCHESTER)

Vibration Dampers. In class (*a*) is the Lanchester torsional vibration damper (Fig. 15), which was probably the first device of this type ever to be used. It consists of a flywheel *A* normally driven by means of spring-loaded friction surfaces *B* from the crankshaft

C, on which the damper is mounted. For maximum effect from a given size of damper it is fitted at an antinode: for the single-node vibration of a multi-cylinder engine this is at the opposite end of the crankshaft to the engine's flywheel.

So long as the frequency and amplitude of angular vibration at the point of attachment of the damper are such that its flywheel can be made to partake of them without exceeding the torque limit for which the friction surfaces are loaded, the damper is "solid" and does not influence the severity of torsional vibration at critical speeds. When higher frequency or greater amplitude at the damper flywheel would require greater peak torques than the friction surfaces can transmit, relative slip takes place first in one direction and then in the other, and the loss of energy so caused sets a limit to the amplitude of vibration of the shaft.

The Sandner damper is similar in basic principle to the Lanchester damper, but the flywheel has internal gear teeth which mesh with those of a number of pinions rotatable about spindles carried by the hub of the damper. Each pinion, in co-operation with the internal gear, acts as an oil pump discharging against a loaded relief valve. So long as the vibratory torque does not exceed an amount corresponding to the setting of these valves the flywheel rotates in unison with the shaft and the damper is inoperative. Otherwise the relief valves permit discharge during certain parts of each vibration cycle, and the resultant loss of energy restrains the building-up of resonant vibration. There are separate relief valves corresponding to the two directions of relative rotation, and by giving them different settings the amount of slip of the flywheel is greater in one direction than in the other. The resultant creep is beneficial in distributing the wear on the gear teeth.

The energy dissipated by a damper of the Lanchester or Sandner type depends on the torque at which it is set to slip. If this is large, there is little slipping (or none at all), and energy dissipation is small. If the slipping torque is small, the energy dissipation is small on that account, even with considerable slip. There is a certain optimum slipping torque which gives maximum dissipation of energy per cycle.

If I = moment of inertia of damper flywheel,
n = frequency constant of vibration to be damped,
λ = amplitude of angular displacement at damper—

$$\begin{array}{l}\text{Damper slipping torque for}\\ \quad\text{maximum dissipation}\end{array} = 0{\cdot}45\ in^2\lambda$$

$$\begin{array}{l}\text{Maximum energy dissipated}\\ \quad\text{per cycle}\end{array} = 1{\cdot}273\ in^2\lambda^2$$

The same expressions apply to the Sandner damper when arranged to slip equally in both directions. By sufficiently loading one set of relief valves, this type of damper may be arranged to slip in one direction only. In that case the optimum slipping torque in that direction is $0{\cdot}32\ in^2\lambda$, and the energy dissipated per cycle is then $0{\cdot}8\ in^2\lambda^2$.

Design of Torsional Vibration Damper. Let the damper be applied at a point whose amplitude of vibration is $T\lambda_o$, where λ_o is that at the reference point O. The procedure in determining the dimensions of the damper is then—

1. Determine from the normal elastic curve the value of λ_o corresponding to the desired maximum stress in the system;
2. Determine from equation (7) the energy input per cycle at this amplitude of vibration;
3. Equate this to the dissipation per cycle when the damper is operating at 80 per cent of its optimum rate, i.e. $0{\cdot}8 \times 1{\cdot}273\ In^2(T\lambda_o)^2$ for two-directional slipping. Hence determine the value of I, the moment of inertia of the damper flywheel;
4. Determine the dimensions of damper springs capable of producing a slipping torque of at least

$In^2T\lambda_0$, i.e. capable of preventing slip when the amplitude of vibration is λ_0;

5. Design the details so that by adjustment of spring loading the slipping torque may be varied down to $0{\cdot}3\ In^2T\lambda_0$;

6. Determine the working value of the slipping torque by adjustment of the springs after trials in service.

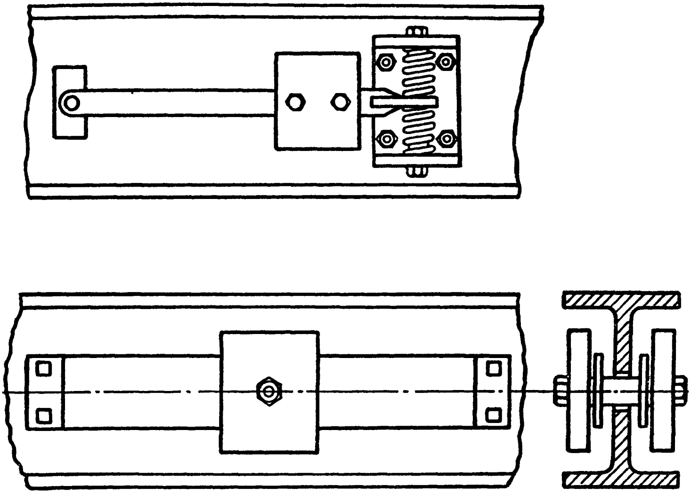


FIG. 16. VIBRATION ABSORBERS ATTACHED TO GIRDERS

Vibration Absorbers. The operation of the dynamic vibration absorber depends on the fact that the influence of a simple harmonic exciting force or torque exerted on a vibratory system may be nullified by attaching to the system at the point of application of the exciting force or torque an auxiliary single-mass, single-spring system designed so that its natural frequency when attached to an infinite mass is equal to the frequency of the exciting force or torque. It may be shown that in such an arrangement the force or torque in the elastic element of the auxiliary system is equal and opposite to the force or torque applied to the main system, which is thus relieved of the original disturbing influence. The auxiliary system (called a *tuned absorber*) must be designed to take the external force or torque continuously.

A form of vibration absorber designed for attachment to girders in structural work was proposed by Professor C. E. Inglis in 1933 and is illustrated in Fig. 16. It consists of a weighted arm lying parallel with the girder, pivoted at one end and flexibly anchored at the other end by springs adjusted to make the natural frequency equal to that of the chief vibration to which the girder is subjected. In another form a weight is attached to a flexible plate of rectangular shape, attached to the girder at its ends but free to vibrate elsewhere.

Such devices have been found effective even when their weight has been as little as 1 per cent of the girder and its load.

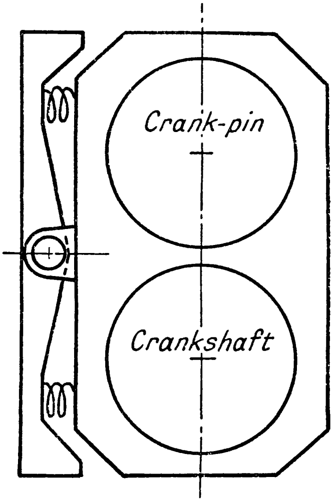


FIG. 17. TORSIONAL VIBRATION ABSORBER ATTACHED TO CRANK WEB

In the case of torsional vibration in a reciprocating engine, an absorber of this type may be attached to each crank and tuned to take any selected harmonic component of the applied torque at a selected speed. Fig. 17 illustrates diagrammatically the arrangement of such an absorber. The additional mass is pivoted on the crank web, and springs exert a restoring torque on it when it rotates out of the equilibrium position.

The Wright Absorber for Torsional Vibration. A particular form of the undamped absorber is the extremely ingenious Wright absorber, in which the elastic element is the centrifugal force in the mass. As this is proportional to the square of the speed of rotation, the natural frequency of the absorber is proportional to the speed, and it consequently absorbs the harmonic torque components of any selected *order* (i.e. multiple of the speed) at all speeds.

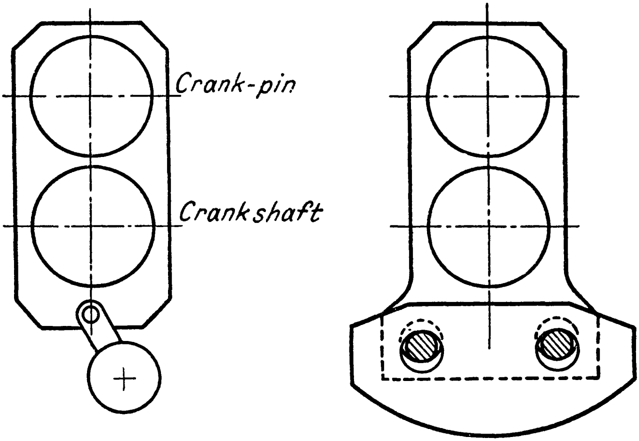


FIG. 18. PRINCIPLE AND ARRANGEMENT OF WRIGHT TORSIONAL VIBRATION ABSORBER

The left-hand diagram in Fig. 18 illustrates the principle of this type of torsional vibration absorber. An auxiliary mass (represented by the lowermost circle) is pivoted on the crank web, and under steady running conditions the link lies radial to the axis of the crankshaft. A disturbance of the mass from the equilibrium position to one such as is shown in Fig. 18 causes the centrifugal tension in the link to have a circumferential component, which exerts a clockwise torque on the crankshaft, just as is the case in the spring-controlled absorber shown in Fig. 17.

In order to tune the Wright absorber to the relatively high natural frequencies which occur in, say, aero engines, the radius of gyration of the auxiliary mass about its pivoting point must be small compared to the dimensions of the attachment. With the normal method of pivoting this is often impossible, and the artifice shown in the right-hand diagram in Fig. 18 is adopted. Here the mass is carried on two pins passing through holes in it and in the crank web. The pins are a loose fit in both sets of holes, and the kinematic effect is the same as if the mass were attached to the crank web by extremely short links.

An interesting feature of the Wright absorber is that it also may be constructed as a loosely mounted balance weight, and its addition need make no increase in the total weight of the engine.

If R = distance of point of suspension from axis of rotation,
r = length of equivalent pendulum,
F = amplitude of harmonic torque to be absorbed,
Y = order of harmonic component to be absorbed,
W = weight of suspended mass,
g = gravitational acceleration,
N = shaft speed (r.p.m.)—

Then $r = R/Y^2$

Amplitude of circumferential movement of suspended mass relative to the crank $= \frac{91\cdot2}{N^2} \cdot \frac{g}{W} \cdot \frac{F}{R + r} \cdot \frac{r}{R}$

This equation makes it possible to determine what weight W is necessary to limit the amplitude of movement of the suspended mass to any particular amount at a specific speed. The device absorbs a certain harmonic component of the torque at all speeds, and if that component is of constant magnitude the displacement of the mass is inversely proportional to the square of the speed.

If it is necessary to suspend the mass by the loose-pin method and the radii of pin and hole are r_1 and r_2 respectively, then—

$$r = 2\,(r_2 - r_1)$$

The amplitude of displacement of the mass cannot exceed this amount, and matters are preferably arranged so that it does not exceed about one-quarter of it at any speed in the normal operating range.

Damped Vibration Absorber. A tuned absorber may also incorporate a damping effect, e.g. by embodying a flexible element made of rubber. The damping makes the absorber rather less effective at its tuned frequency than it would be if undamped, but on the other hand the damping causes some reduction in amplitude of vibration at frequencies other than the tuned frequency. Hence the damped absorber is not essentially inferior to the undamped one, and it permits the use of rubber as the flexible element. This may appreciably facilitate design.

A damped absorber which has been applied to torsional vibration consists essentially of a cylinder of rubber which acts both as the mass and the flexible element. The natural frequency for torsional vibration is $(14\,700/l)$ cycles/min., where l is the length in inches of a cylinder supported at one end or half the length of a cylinder supported at both ends. The necessary diameter of the cylinder depends only on the torque to be absorbed and the permissible shear stress in the rubber. This is of the order of 30 lb./in.^{2}

Damping of Vibrations by Impact. In certain cases damping of vibrations has been effected by making use of the dissipation of energy which occurs on impact. A weight is loosely mounted in the vibrating member and is consequently struck by it twice in each cycle. There is a certain optimum ratio of clearance to amplitude, and this must be found by trial.

In reference 26 a description is given of the application of this method to the damping of vibration in steam turbine buckets.

Pins lightly fitted into holes in the rim of a gearwheel have been found to reduce appreciably the noise made by the gear when running in mesh with another gear.

In reference 38 details are given of a torsional vibration damper which works partly by viscous dissipation of energy and partly by impact loss. An oil cushion is used to prevent metallic contact between surfaces which would otherwise meet with impact, so that the action is less severe than would be the case in a true impact-damper of the same dimensions.

CHAPTER VI

TRANSVERSE VIBRATION OF SHAFTS

VIBRATION induced in shafts by forces acting perpendicular to them are in general much less troublesome than torsional vibrations. Consideration of a multicylinder internal combustion engine will indicate broadly why this is the case. Here the transverse forces applied to the crankshaft by the connecting rods are considerable, but by using main bearings extended right up to the crank webs, each throw is well supported and is thus stiff enough to preclude any possibility of resonant vibration. On the other hand, such construction does not stiffen the shaft in the torsional sense; that can be done only by an increase in diameter of journals and crankpins.

Generally speaking, transverse vibration of shafts occurs in serious degree only as the effect of unbalance in masses attached to the shafts in conjunction with resonance established when the speed of the shaft is equal to a natural frequency of transverse vibration. A critical speed of this nature is usually extremely high unless the shaft is slender or carries considerable weight.

For example, the lowest critical speed of a uniform steel shaft of diameter d, supported in short bearings distance l apart, is 4 800 000 d/l^2 r.p.m. With 2 in. diameter and 36 in. bearing span (for example), this works out at 7 400 r.p.m.

With modern methods of balancing rotating parts the accuracy of balance obtained is so high that no excessive vibratory loads on bearings need be expected unless a critical speed is attained. In this case the magnification due to resonance can produce dangerous vibration, even in shafts which are in the ordinary sense perfectly balanced.

Natural Frequencies for Transverse Vibration. The natural frequency of transverse vibration of a beam carrying a single load may be determined by use of equation (4) (page 6). If there are several concentrated loads, a closely approximate value for the lowest natural frequency is given by Dunkerley's formula—

$$\frac{1}{N^2} = \frac{1}{N_1^2} + \frac{1}{N_2^2} + \frac{1}{N_3^2} + \text{etc.} \quad . \qquad . \quad (9)$$

where N is the natural frequency of the whole assembly, N_1 is the natural frequency when the shaft carries only mass No. 1, N_2 when it carries only mass No. 2, and so on.

The same methods of calculation apply to a revolving shaft, provided that the masses it carries are of small transverse dimensions compared with the length of the shaft. Otherwise a correction has to be applied on account of the bending effect exerted on the shaft by equal and opposite centrifugal forces which are not in line with each other when the shaft bends. When the speed of rotation is equal to the natural frequency this correction can be simply expressed.

Formulae are given in Table III for natural frequencies of transverse vibration of a number of typical systems.

For more complicated cases the frequency may be calculated by a general method based on the principle of equating maximum kinetic energy to maximum strain energy. The normal elastic curve or "deflection curve" is first assumed to be the same as the deflection curve of the shaft under static loads proportional to the attached masses.

Let the weights and deflections be W_1, W_2, etc., y_1, y_2, etc. Then—

$$\text{Total strain energy} = \tfrac{1}{2}\,(W_1y_1 + W_2y_2 + \text{etc.})$$

If the natural frequency is N cycles/min. the maximum vibration velocity of mass No. 1 on passing through the equilibrium position is $\pi N y_1/30$, and the kinetic energy of that mass is $(\tfrac{1}{2}\, W_1/g)(\pi N y_1/30)^2$.

The maximum total kinetic energy is equal to the maximum total strain energy, and hence it is found that—

$$N = 188\sqrt{\left(\frac{\Sigma W_1 y_1}{\Sigma W_1 y_1^2}\right)} \text{ cycles/min.} \quad . \quad . \quad (10)$$

This method is not exact, because the maximum deflecting load exerted by any vibrating mass is proportional to the deflection as well as to the mass. Hence the normal elastic curve does not coincide with the static deflection curve. The difference has fortunately only a small effect on the calculated frequency, and the method is sufficiently accurate in practice for the determination of the lowest natural frequency of transverse vibration of a shaft carrying loads at points between the bearings. If there are overhung loads, the method is less accurate.

The values y_1, y_2, etc., to be used in equation (10) are calculated from the loads on the shaft, the dimensions of the shaft, and the manner of its support by any of the well-known methods. In general, and especially if the shaft is of non-uniform section, the graphical method using the funicular polygon is most convenient.

It must be emphasized that there is no essential connexion between static deflection and the frequency of natural vibration, but the possibility of determining one from the other is useful because methods of calulating static deflections have been well established for other reasons.

The natural frequency is independent of the angular position of the shaft in relation to the horizontal. A vertical shaft has no transverse static deflection by virtue of the loads it carries; nevertheless its natural frequency is the same as if it were horizontal.

Example. Let it be required to calculate the lowest natural frequency of transverse vibration of a shaft which carries loads of 800, 1 000, and 700 lb. at points situated at known distances from the bearings.

Under the influence of these loads the shaft has a certain deflection curve, which is determined by any convenient method, and from which the deflections at the loads themselves are found to be 0·0114, 0·0162, and 0·0146 in. respectively. Then—

$$\begin{aligned}\Sigma W_1 y_1 &= \ \ 800 \times 0{\cdot}0114 = 9{\cdot}12\\ &\quad + 1\,000 \times 0{\cdot}0162 = 16{\cdot}2\\ &\quad + \ \ 700 \times 0{\cdot}0146 = 10{\cdot}22\\ &\qquad\qquad\qquad\qquad\ \overline{35{\cdot}54}\end{aligned}$$

$$\begin{aligned}\Sigma W_1 y_1^2 &= \ \ 800 \times 0{\cdot}0114^2 = 0{\cdot}104\\ &\quad + 1\,000 \times 0{\cdot}0162^2 = 0{\cdot}262\\ &\quad + \ \ 700 \times 0{\cdot}0146^2 = 0{\cdot}149\\ &\qquad\qquad\qquad\qquad\ \overline{0{\cdot}515}\end{aligned}$$

and, using equation (10)—

$$N = 188\sqrt{\frac{35{\cdot}54}{0{\cdot}515}} = 1560 \text{ cycles/min.}$$

On account of the phenomenon of whirling, severe transverse vibration will be set up at this speed, and the normal running speed should not be between about 1 200 and 2 000 r.p.m.

Whirling of Shafts. A shaft supported in two bearings and carrying a single central load in the form of a disk adjusted so that its centre of gravity lies in the shaft's axis produces severe vibration, known as *whirling*, at and near a certain critical speed. This speed is found to coincide with the natural frequency of transverse vibration of the shaft and disk, but the exact nature and causes of the vibration cannot easily be explained in a manner that leads to universal acceptance.

The essential practical point is, however, that the running range of speed must not include such a critical speed. The speed is identical with the natural frequency of transverse vibration for any shaft arrangement.

Very often it is a speed much higher than the operating speed, but in general it is necessary to calculate the natural frequency of transverse vibration of the shaft in the design stages in order to make sure that no sustained running speed is within about 30 per cent of a critical speed. Sometimes the running speed is higher than a critical speed, and in such cases there is momentary whirling during starting up and shutting down.

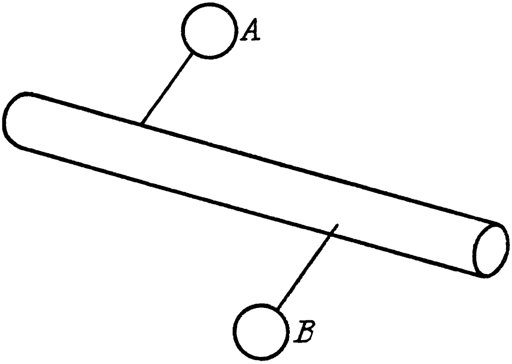


FIG. 19. ROTATING PARTS STATICALLY BALANCED BUT DYNAMICALLY UNBALANCED

Balancing Machines. Fig. 19 shows a uniform shaft carrying two equal masses A and B on equal arms extending in opposite directions from different points on the axis of the shaft. The centre of gravity of the assembly lies on the axis of the shaft, and no static test will detect any unbalance. If, however, the shaft is rotated at high speed in bearings supporting it near its ends, the centrifugal forces on A and B produce a couple acting in a rotating plane, with the result that each bearing is subjected to a load whose direction constantly changes, with the effect of severe vibration. The system is said to be *dynamically unbalanced*, and the detection and correction of unintentional unbalance of this nature is usually carried out in a special machine.

This consists essentially of two bearings, flexibly mounted but capable of being locked, and a motor arranged to rotate the part to be tested while mounted in the bearings. The connexion between motor and work is by means of a quickly disengageable dog clutch. The bearings are locked, and the motor and work are run up to full speed. The clutch is disengaged, one bearing is unlocked, and the motor brought to rest. The speed of the work gradually diminishes because of windage and bearing resistances, and the flexibility of the bearing mounting is such that a critical speed is passed through before the work comes to rest.

The amplitude of vibration at the critical speed is recorded by pencil on paper, and another pencil pressed against a journal of the work records on it the positions of the high spot between the top speed and the critical speed. A similar test in the opposite direction of rotation gives different positions of the high spot. At speeds above the critical speed the high spot and the heavy spot are theoretically separated by 180°, while at resonance the separation is 90°. (See section on Magnification and Phase Difference.) Owing to damping effects and friction in the mechanism, actual markings do not usually coincide exactly with these expectations, but the agreement is close enough to indicate the position of the heavy spot.

With this information a piece of Plasticine of arbitrary weight is added to the work at some convenient point opposite to the location of the heavy spot, and the procedure is repeated several times. When this has succeeded in reducing the amplitude of vibration at resonance to an amount that the machine cannot record with certainty, the same procedure is carried out with only the other bearing free.

It does not necessarily follow that, even after this, the work will be in balance with *both* bearings free, but a simple calculation taking into account the positions and weights of the Plasticine already added shows where to add a third piece of Plasticine to secure true dynamic balance within the limits of sensitivity of the machine.

To complete the balancing operation, pieces of metal are removed from the work (or added to it) in such a way as to duplicate the dynamic effects of the Plasticine, which is then removed.

An alternative method of determining the position of heavy spots is to run the work at any conveniently high speed in any suitable bearings and to measure the amplitude of vibration at each bearing by means of a sensitive *vibrometer*. A very suitable type of instrument for this purpose is an electrical one which, in conjunction with a neon lamp and a contact-maker driven by the work, shows the high spot on the work in relation to a series of numbered points previously marked on it.

A stylus pressed against the bearing housing transmits its vibrations to a mirror, which rocks in unison and swings a ray of light backwards and forwards over a scale. At high frequencies this gives the appearance of a continuous light-band, whose length is a measure of the amplitude of vibration of the bearing housing. A ray of light from a small neon lamp flashing in unison with that which illuminates the work is also reflected by the mirror, and the position of the resulting red spot in relation to the light-band on the scale indicates the phase relation between the bearing vibration and the setting of the contact-maker. By adjustment of the contact-maker the neon lamp can thus be made to flash when the vibrometer is reading the high spot.

Procedure in Dynamic Balancing. Fig. 20 represents a rotor which is to be dynamically balanced by the addition of weights in the selected planes *I* and *II*. The procedure is—

1. Lock bearing *B* and find by repeated trial how to add weight in plane *I* to eliminate vibration at bearing *A*.

2. Lock bearing A, unlock bearing B, and eliminate vibration at B by adding weight in plane II. Let this weight be W at radius R.

3. Add to the weight W a weight $\dfrac{Wac}{b(a+b+c)}$.

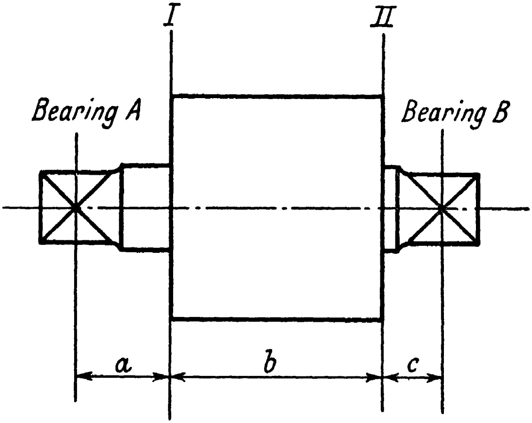


FIG. 20. ILLUSTRATING PROCEDURE IN DYNAMIC BALANCING

4. In plane I place a weight $\dfrac{W(a+b)c}{b(a+b+c)}$ at radius R on the opposite side of the axis to the weight in plane II.

5. The two weights now in plane I may be replaced by a single weight, equal to their dynamic resultant, in that plane.

Steps 3, 4, and 5 in this procedure avoid the repeated rebalancing (first with one bearing locked, then with the other) which is otherwise necessary. Such unsatisfactory trial-and-error practice is sometimes adopted by those unaware of the direct method detailed above.

CHAPTER VII

USE OF FLEXIBILITY TO DIMINISH VIBRATION

It is well known that shock due to impact may be reduced by the introduction of spring-like members. For example, railway vehicles have spring buffers to reduce compressive shocks, and spring drawbars to reduce tensile shocks. The success of devices of this character is associated with the fact that a spring absorbs a relatively large amount of strain energy without excessive loads, whereas the capacity of rigid metal members for strain energy is low. (To illustrate this it may be mentioned that a strain energy of merely 10 ft. lb. in a steel tension member 3 in. in diameter and 10 in. long involves a tensile stress of nearly $4\frac{1}{2}$ tons/in.^{2}.) Any additional springiness or flexibility, however small, is of some benefit in reducing stresses due to impact.

This has given rise to the erroneous impression that flexibility is necessarily advantageous in reducing vibration. Actually this is not the case, and it must be emphasized that the indiscriminate use of flexibility is as likely to *increase* vibration as to reduce it.

The general truth of this statement will be appreciated after a review of Fig. 4 (page 10). It will be seen that the magnification is greater than unity so long as the ratio of frequency of external force to the natural frequency of the system lies between 0 and $\sqrt{2}$, or in other words so long as the natural frequency of the system lies between infinity and 0·71 times the frequency of the exciting force. No change of flexibility that fails to move the natural frequency outside this range is certain to be advantageous. If the change happens to make the natural frequency approach the exciting frequency, the change will lead to more serious vibration. In general there is a certain minimum degree of flexibility which is beneficial, and the greater the amount by which this minimum is exceeded the more nearly is complete elimination of vibration effected.

This is the guiding principle in the design of elastic supports intended to diminish transmission of vibration. For example, an engine may develop vibratory forces (due to unavoidable imperfections in balance) with a frequency of 500 cycles/min. If it is desired to prevent the transmission of undue vibration to the floor and thence to other parts of the building in which the engine is situated, this may be done by elastically mounting the foundation block. If, however, the arrangement is to be effective in reducing transmission of vibration below that which would occur with completely rigid mounting, the natural frequency of vertical vibration of the engine and its foundation block must be lower than 350 cycles/min., and to gain any appreciable advantage from the elastic mounting the natural frequency must be considerably lower than this, say 120 cycles/min.

There are thus two essential considerations in the design of the flexible element which forms the spring mounting. They are—

1. That it must be strong enough to carry the static load due to weight of engine and foundation block together with the maximum vibratory force transmitted to the main foundation;
2. That its stiffness must be low enough to bring the natural frequency well below 0·71 times the frequency of the vibration whose effect is to be reduced.

These necessities sometimes combine to make a metallic mounting impracticable by reason of inconvenient dimensions, and in practice much use is made of rubber, cork, and felt for this purpose. The low mechanical strength of these materials requires that they be used in sheets, so that the load is spread over a large area; but their low moduli of elasticity permit of an adequate degree of flexibility without excessive thickness.

In starting up or shutting down a machine mounted in this way it is necessary to pass through a critical speed at which resonance with the natural frequency of the spring-mounting is momentarily established. Provided that the acceleration in the neighbourhood of that speed is sufficiently rapid, there is no opportunity for dangerous vibration to be produced. The use of rubber, cork, or felt is also advantageous in this connexion, as those materials have very pronounced damping qualities by virtue of internal friction, and consequently they keep down the amplitude of the resonant vibration to a much smaller amount than would occur with a metallic spring element.

From equation (4) it may be deduced that when the natural frequency of the mounting is less than about one-third of the frequency of the exciting force, the fraction of that force transmitted to the floor is approximately equal to $\left(\frac{\text{natural frequency}}{\text{exciting frequency}}\right)^2$. For example, a frequency ratio of 4 gives a vibration reduction or attenuation of about one-sixteenth.

Spring-mounting of Machines. A very effective form of spring-mounting is that shown in Fig. 21. The foundation block is suspended by a number of steel rods in tension from a series of vertical steel springs in compression, while other springs placed horizontally control side movement of the block. In designing this type of mounting consideration has to be given to the various modes of vibration (both linear and angular) of the suspended mass. Self-contained spring-mounting members are sometimes described as *anti-vibrators.*

Spring supports may be used to reduce transmission of vibration both from the suspended mass to the floor and from the floor to the suspended mass. The second facility is important in designing the mounting of sensitive instruments in buildings subjected to vibration.

Magnification in Flexible Support with Damping. The magnification whose variation is expressed by the curve in Fig. 4 is the ratio of amplitude of force in the spring to that applied to the mass, when the latter is subjected to some external damping influence. Damping restrains the vibration by an alternating force of the same frequency as the exciting force, and if the reaction of the damping force is taken by the member which supports the spring, the total force on that member is of greater amplitude than that in the spring. The modified expression for magnification factor, which may in this case be called the "transmissibility," is—

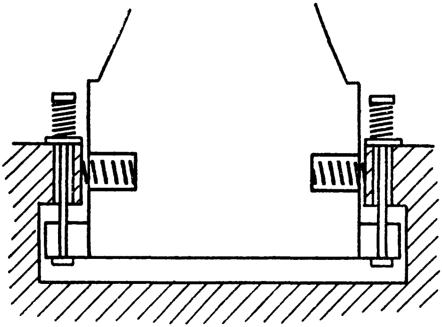


FIG. 21. METHOD OF SPRING-MOUNTING A MACHINE

$$M = \sqrt{\left[\frac{(M_0^2 - 1) + (p/n)^2}{(M_0^2 - 1)\,[1 - (p/n)^2]^2 + (p/n)^2}\right]} \quad . \quad (11)$$

where M_0 is the transmissibility when the frequency of the external force is equal to the natural frequency of the system in the undamped condition. M_0 is very nearly the maximum transmissibility. As before, it is equal to—

$$2\pi\left(\frac{\text{maximum strain energy of system}}{\text{energy loss per cycle by damping}}\right)$$

Insertion of a few numerical values in equation (11) will show that the introduction of damping into a flexible support diminishes the amplitude of vibration in the resonance condition, but actually *increases* it above the value achieved by the undamped flexible support at frequencies much higher than the natural frequency. The increase is usually very small and is outweighed by the advantage of diminished vibration in the condition of resonance. This is specially important if there is much starting and stopping and consequent passages through resonance.

It is necessary to remember that a spring-mounted machine has various modes of vibration. Although the vertical linear vibration comes most naturally to mind, horizontal vibrations are possible, and fluctuating torques may produce angular vibrations about various axes.

Spring-mounting of Vehicles. The fundamental principle to be observed in the design of vehicle springs is that of making the natural frequency of vibration very low and thus reducing the transmission of high-frequency vibration and shocks. This means comparatively flexible or "soft" springs.

There are, however, other considerations which prevent the use of extremely soft springs. The variation in deflection of the spring between light loading and heavy loading of the vehicle sets a limit to practicable softness. In a road vehicle extreme flexibility may lead to imperfections in road-holding and controllability on curves.

It is not difficult, in general, to design a vehicle mounting with natural frequencies much lower than that of vibration caused by normal road roughnesses, but it is possible for resonance to be established by running at a particular speed over a regular wave formation of road surface.

To avoid the unpleasant and possibly dangerous vibration that would otherwise be set up in such circumstances, and also to extinguish quickly the natural vibrations which follow an isolated shock, some damping arrangement is necessary. This is usually in the form of so-called "shock absorbers." Actually the name is misleading, because such devices tend to make the springing less satisfactory as a protection against shocks.

The common type of automobile shock absorber is a hydraulic dashpot. A cylinder is connected to the axle and contains a piston connected to the chassis, and movement of the piston in the cylinder causes oil to pass from one side of the piston to the other through restricted openings. Viscous resistance to the flow of the oil opposes the motion of the piston, and this provides the desired damping.

These remarks refer particularly to vertical vibrations of the vehicle on its springs. There are, however, six possible modes of vibration of a mass spring-borne in the way common in vehicles. They are—

1. Vertical vibration without change of angular position.
2. Fore-and-aft vibration without change of angular position.
3. Side-to-side vibration without change of angular position.
4. Angular vibration about a vertical axis.
5. Angular vibration about a horizontal fore-and-aft axis.
6. Angular vibration about a horizontal transverse axis.

Any combination may occur at the same time.

Of these only 1, 5, and 6 need usually be considered, natural frequencies in the other modes of vibration being normally very high.

The natural frequency in each mode of vibration can be determined from the spring stiffnesses, the masses of the various parts, and their moments of inertia about the appropriate axes passing through their centres of gravity.

Thus, if the spring stiffness of each front spring is s_1 and of each rear spring s_2, and the weight of the vehicle is W, and its moments of inertia about AA and BB are I_A and I_B, then, referring to Fig. 22—

Natural frequency for vertical vibration

$$= 376\sqrt{\left[(b+c)\Big/W\left(\frac{c}{s_1}+\frac{b}{s_2}\right)\right]} \quad . \quad . \quad (12)$$

Natural frequency for angular vibration about AA

$$= 266a\sqrt{\left(\frac{s_1+s_2}{I_A}\right)} \quad . \quad . \quad . \quad . \quad (13)$$

Natural frequency for angular vibration about BB

$$= 266\sqrt{\left(\frac{s_1 b^2+s_2c^2}{I_B}\right)} \quad . \quad . \quad . \quad (14)$$

If each assembly of axle and two wheels be regarded as a separate mass, the number of possible modes of vibration of the complete vehicle is 18. In practice the additional modes to be considered are confined to—

1. Vertical vibration of each wheel-and-axle assembly with little chassis movement.
2. Angular vibration of each wheel-and-axle assembly about a fore-and-aft horizontal axis with little chassis movement.

Use of Rubber as an Elastic Element. During comparatively recent years there has been extensive development in engineering practice of the use of certain compounds of rubber. Its low elastic moduli and its high hysteresis loss are often valuable characteristics when it is used as the flexible element in spring-mountings and the like.

It may be used in tension, compression, or shear. The first of these is not usually advisable, because rubber in tension is easily cut and the rupture rapidly spreads; furthermore, it is not easy to arrange neat attachments for rubber in tension. When used in compression, rubber bulges laterally and friction is thereby set up between its loaded surfaces and the metal ones with which they make contact. This may lead to local abrasion of the rubber.

The most successful application of rubber in compression or shear is dependent on the "bonding" of rubber and metal. By a special vulcanizing process,

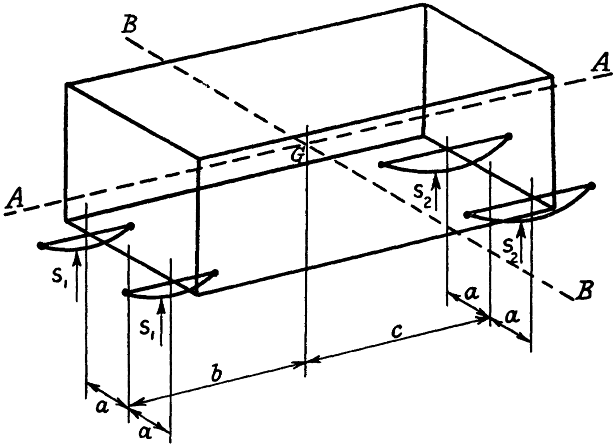


FIG. 22. SPRINGING OF FOUR-WHEEL VEHICLE

rubber and brass-plated steel can be united in a bond which is stronger than the rubber itself. As the application of this process imposes some limitations on the shapes of the metal members, it is common practice to bond the rubber to two thin metal members conveniently shaped for that purpose and so to form a "rubber sandwich." This is regarded as the flexible element, and the adjoining parts of the machine are designed to accommodate it.

The simplest form of sandwich is shown in Fig. 23 (*a*). A combination of two of them makes a second form (Fig. 23 (*b*)). A third type, shown in Fig. 23 (*c*), is a development of the second one in that the outer members have been extended to meet each other, thus encircling the inner member; all are of circular section. This type may take axial force, transverse force, torque about the axis, or any combination of these loadings.

In applications of this type, the rubber acts not only as the flexible element but also as a guide for the movement of one metal member relatively to the other. For example, in the sandwich shown in Fig. 23 (*b*), the rubber blocks allow the central plate to vibrate vertically under the action of vertical forces, but restrain any tendency to wander horizontally.

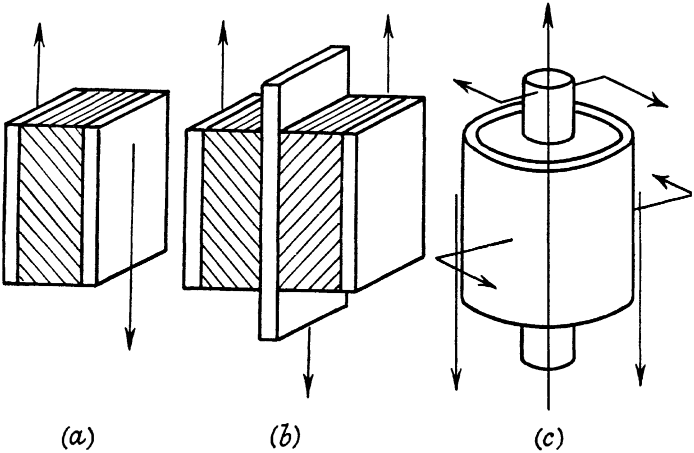


FIG. 23. RUBBER SANDWICHES

Rubber mounting of automobile engines is now very common, and the same material is also used in mounting electric motors where vibration and noise must be minimized. In many cases the rubber must be specially shaped to give flexibility in some directions and comparative rigidity in others.

Rubber is also very effective in restricting the transmission of audible vibrations. This is partly due to its heavy hysteresis loss, which for a cycle of reversed stress is of the order of one-half the maximum strain energy.

Use of Cork in Spring-mountings. As a substitute for steel springs or rubber sandwiches, cork in blocks reinforced by iron bands may be used as a compression spring for supporting machinery. A suitable form of cork block has a plan view which is a square of side equal to about twice the height. As in the other cases, a low natural frequency of the system is necessary if transmission of vibration to the earth is to be reduced.

The compressive stiffness of cork varies with stress, thickness of block, and frequency of vibration. It should be designed for a static compressive stress not exceeding about 25 lb./in.^{2}, and an actual test on a block of the proposed dimensions is desirable to determine what the stiffness will be under running conditions. It is of the order of 1 500 lb./in.^{2} per inch of compression per inch of thickness, i.e.—

$$\begin{array}{c}\text{Change in}\\ \text{stress}\end{array} = \frac{1\,500 \times \text{Change in thickness}}{\text{Thickness}} \text{ lb./in.}^2$$

CHAPTER VIII

MEASUREMENT OF VIBRATION

To investigate difficulties due to vibration in machinery it is necessary to have some quantitative method of recording vibration. Instruments for this purpose are divided into three main classes—

1. Those which measure frequency;
2. Those which measure amplitude;
3. Those which indicate the nature of the wave form of the vibration in addition to measuring frequency and amplitude.

Measurement of Frequency. In the first class the simplest device is a flexible reed of adjustable length. This is clamped at its "fixed" end to the part of the machine under consideration, and while the machine is running at constant speed, the length of the reed is adjusted to show the maximum amplitude of vibration at its free end. This occurs in the condition of resonance, the natural frequency of vibration of the reed (which is easily determinable from its length) then coinciding with the frequency of vibration of the point of attachment of the instrument. The Fullarton vibrometer is an instrument of this class.

Measurement of Amplitude. In the second class of instrument there are two essential requirements—

1. A stationary member to form a reference with which the vibratory movements may be compared;
2. A means of multiplying the vibratory movement so that it can easily be measured.

If the vibrating assembly is large, it may not be easy to find any adjacent point of support that is free from vibration, and accordingly the usual method of providing a stationary reference point is to mount a heavy mass on springs arranged to give a frequency of natural vibration which is low compared with that of the vibration to be investigated. By this means, transmission of vibration to the suspended mass is reduced to a negligible amount, and the mass remains stationary.

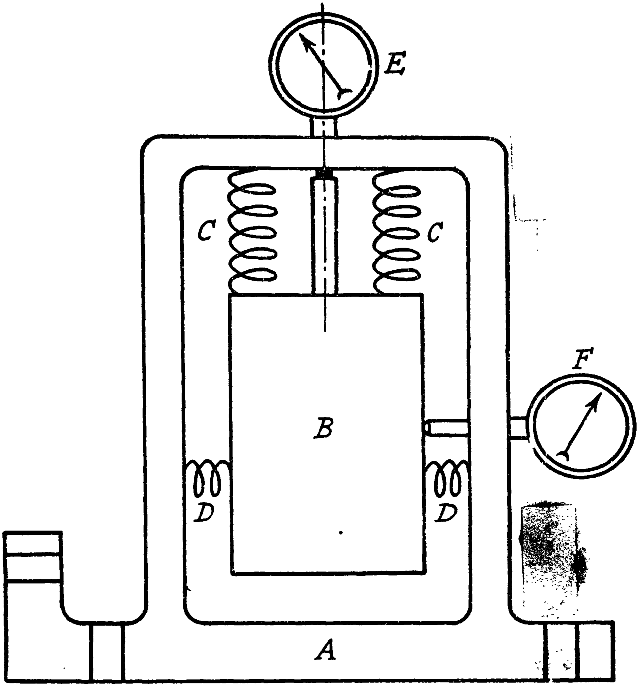


FIG. 24. VIBROGRAPH

Fig 24 shows diagrammatically the construction of such a vibrograph. It consists of a frame A which is attached to the member under investigation, and a heavy mass B suspended from A by springs C.

Lateral control of B is effected by springs D, and the natural frequency in both vertical and horizontal directions is about 200 cycles/min. In operation, A

partakes of the motion of the vibrating part while *B* remains stationary. The necessary multiplying mechanism is contained in dial indicators *E* and *F*, which measure the vertical and horizontal relative vibrations of *A* and *B*, and since *B* is stationary these are the vibratory movements of the member to which *A* is attached. The double amplitudes of vibration are shown by the arcs through which the indicator needles move.

Another method of observing the vibrations is to focus a microscope, attached to the main body of the instrument, on a brightly illuminated piece of emery cloth glued to the heavy mass. By adjusting the position of the light source, one or more crystals in the emery cloth may be made to shine in the field of the microscope, and its motion becomes apparent as a bright line in the form of a loop. If the shape of the loop is constant, it is an indication of vertical and horizontal vibrations of equal frequency and constant amplitude.

A fundamentally similar instrument, confined to the measurement of vertical vibrations, embodies, instead of a dial indicator, a lever multiplying mechanism operating a stylus which records the amplitude of vibration on a moving strip of celluloid. Another record is simultaneously made on the celluloid by a time-marking stylus. When the strip is examined under a microscope, both the amplitude of vibration and its frequency can be accurately measured.

The same general principle applies to all the various forms of torsiograph. Instruments of this class were first devised to measure amplitudes of angular vibration from which torsional vibration stresses could be deduced. For this reason, no doubt, the name "torsiograph" was applied, but it is misleading inasmuch as the instrument does not directly measure twist.

The torsiograph (Fig. 25) consists of a light pulley, driven through a belt or otherwise by the shaft whose angular vibrations it is required to measure. Care is taken, by lightening the pulley and by using a connexion of minimum practicable elasticity, to ensure that the angular movements of the pulley are accurately proportional to those of the shaft. Coaxial with the pulley is a heavy flywheel, and the two are connected by a spiral spring which is very flexible. The flywheel corresponds to the heavy mass in the vibrograph and runs at constant speed, because the flexibility of the spiral spring prevents the transmission of high-frequency angular vibration. The relative angular movement of pulley and flywheel is converted by suitable mechanism into axial movement of a non-rotating rod which carries a pen or stylus recording the angular vibrations on a moving strip of paper or celluloid.

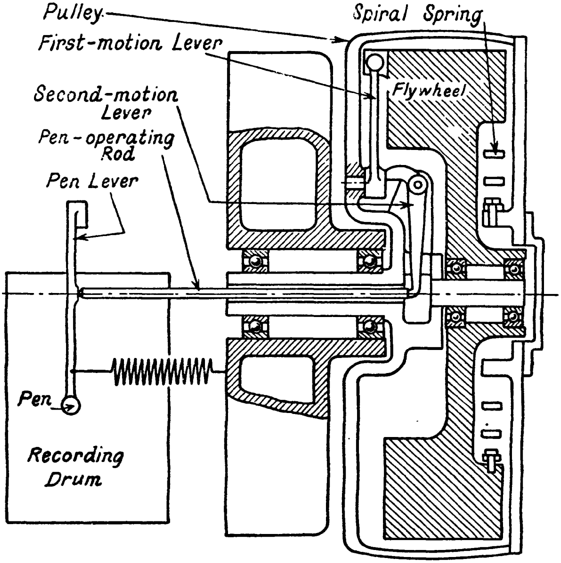


FIG. 25. GEIGER TORSIOGRAPH

Accelerometers. The instruments described above make a direct measurement of amplitude of linear or angular vibration. In the *accelerometer* the construction is basically similar, but the mass and the elastic element are such as to give a natural frequency much *higher* than that of the vibration to be observed. The relative movement of the ends of the elastic element is a measure of the amplitude of *force* transmitted through it, and therefore of the force exerted on the part of the instrument carried by the elastic element. (The latter is a consequence because the applied vibration is of relatively low frequency and the transmissibility is therefore very nearly unity.) Hence the amplitude of acceleration of the moving surface is known, and if its motion is assumed to be simple harmonic, the amplitude of linear vibration is calculable from the frequency.

Oscillographs. Although the recording vibrograph and the torsiograph give all necessary information in respect of frequency and amplitude within their physical limitations, the scale of the record is not large enough to provide any great detail of the wave form of the vibration, even if the instrument were capable of responding accurately to high-frequency components of the vibration. For this purpose the oscillograph, originally developed for investigation of alternating electric current waves, is often applied to mechanical vibrations. Forced vibrations of different kinds often occur simultaneously in machinery, and knowledge of the wave form of their resultant may be necessary.

The type of instrument that has the widest range of application and has at the same time a degree of accuracy much higher than is necessary for most mechanical vibration investigations is the cathode-ray oscillograph. In this the vibration to be measured is converted into linear vibration of one plate of an electrostatic condenser. If the vibration under observation is a linear displacement, it suffices to make a direct connexion between the plate and a "feeler" applied to the vibratory surfaces. If it is desired to record variation of force or, say, gas pressure, the force is applied to an elastic element whose distortion under load causes movement of the plate of the condenser.

The consequent variation in electrostatic capacity is amplified electrically and causes variation in the direction of a cathode ray in such a way as to change the vertical position of the point at which it strikes a fluorescent screen. Horizontal movement (called the *time base*) of the beam is controlled by independent electrical means. It consists of uniform movement at a selected speed over a straight path, followed by a very much more rapid return to the initial position: this cycle is repeated indefinitely. If its frequency is made equal to the fundamental frequency of the vibration under examination, the commencement of a new cycle of that vibration always finds the cathode ray in the same horizontal position. The ray therefore suffers the same vertical displacements during each horizontal traverse, and its intersection with the fluorescent screen thus traces the wave form of the vibration. This can either be traced by pencil on translucent paper or be recorded photographically.

The horizontal and vertical scales of the movements of the cathode ray are steplessly variable while the instrument is working, and the relation between vertical scale and amplitude under observation is determined by a separate direct calculation.

A standard instrument of this type (as made, for example, by Southern Instruments Ltd.) can indicate accurately at frequencies up to 10 000 cycles per *second*. (In special circumstances a cathode-ray oscillograph has indicated electrical vibrations making 220×10^6 cycles per *second*.)

Wave Form of Recorded Vibration. Vibration in any part of a machine is usually compounded of vibrations from different sources, and a sensitive instrument—vibrograph, torsiograph, or oscillograph—will show this by a wave form which is not a pure sine curve. If the frequencies of the component vibrations are not exact multiples of one another, the wave form will continually change. A vibration record taken from a rotating shaft by means of a Geiger torsiograph is shown in Fig. 26.

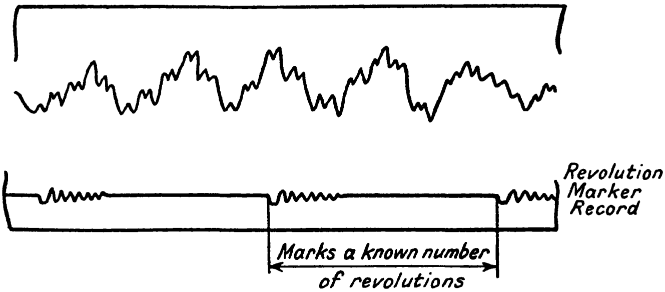


Fig. 26. Typical Record from Torsiograph

If, however, resonance has been established with some harmonic component of a vibration-exciting influence, the amplitude of the resulting vibration may be so much greater than any other that the wave form is substantially a sine curve. This occurs in torsiograph records taken at a critical speed, and measurement of the amplitude of angular displacement associated with the resonant vibration is then easy.

CHAPTER IX

MACHINERY FAILURES DUE TO VIBRATION

In many cases the chief objection to vibration produced by machinery is its effect on human beings who have to remain in its vicinity. Usually it is vibration that shows itself as noise that is most troublesome in this connexion, but vibration of lower frequency is often transmitted from the machine which generates it to other parts of the same building, and may show itself there more prominently than at its source by resonance in some elastic object. Just as the free end of the reed of a Fullarton vibrometer, when adjusted to give resonance, has a much greater amplitude of vibration than has the other end, so may the upper stories of a building vibrate more noticeably than do the lower stories in which the disturbing forces are created. So also may the top of a table be set in severe vibration by resonance with a floor vibration, which in itself is not objectionable.

The human frame is sensitive, qualitatively at all events, to vibration, and in general this is some safeguard against mechanical failure from that cause. (An amplitude of 0·001 in. at more than about 500 cycles/min. is perceptible to the touch, and at audible frequencies much smaller amplitudes can be detected.) Vibration shows itself most effectively as noise when there are loose parts in which repeated impacts are thereby produced. Although noise of this nature offends the mechanical sense and draws attention to the possibility of slackening of fits and fastenings by wear and fatigue, disastrous failure is not necessarily to be expected. For example, gears which are intolerably noisy may continue to operate without any sign of fatigue.

In general, a failure due to vibration occurs only when the vibratory forces have been multiplied beyond normal expectations by the establishment of resonance. Transverse vibration of shafts offers this possibility in certain circumstances, but for many classes of machinery it is possible to avoid critical speeds without adopting any exceptional proportions in design. If, for any reason, this has not been done and a critical speed is reached in service, normal bearing clearances give scope for impacts which show themselves in warning noise.

Severe torsional vibration has more chance of occurring without giving audible warning. An assembly of rotating parts firmly keyed to shafts rigidly coupled together may be suffering large vibratory torques without giving any sign that can be detected without special instruments. Such a vibration can be maintained, however, only by some vibratory external torque, and in practice this means that torsional vibration is rarely serious in installations which do not include cranks and connecting rods. These, with their bearing clearances, provide audible effects which betray severe torsional vibration. Purely rotary machinery has no such safeguard, but on the other hand it is much less likely to be subjected to vibratory torques which could maintain a dangerous torsional vibration.

The phenomenon of torsional vibration has forced itself into prominence in the past by leading to failures in internal combustion engine drives. Unless special precautions are taken in the design of such plant there is a possibility of running at a critical speed for torsional vibration, and in such event failure of some part of a shaft will result if the vibratory stress exceeds the fatigue limit of the material.

Some details concerning certain failures due to resonant torsional vibration are mentioned below.

1. A marine propeller shaft fractured after some twelve months' service in a way which suggested fatigue due to fluctuating torsional shear stress. The stresses calculated from the engine torque diagram on the usual assumption of rigid shafting were well within safe

limits. A second shaft of identical dimensions developed a crack which promised to produce similar failure, and a third shaft gave the same result. Calculation then showed that the normal cruising speed was close to a critical speed for torsional vibration. It was not convenient to change the running speed, and an increase in shaft diameter would have led to considerable expense in several directions. A fourth shaft was therefore made with a diameter some 10 per cent *less* than that of the original shaft, and the remaining parts of the tunnel shafting were reduced to the same figure. This brought the troublesome critical speed below the normal running speed, and no further difficulty was experienced.

2. Four similar Diesel-driven generator sets were installed in certain American ships, and they were put into service at the same time. They were designed to run at a constant speed of 150 r.p.m. After a few weeks' running one of the crankshafts fractured in the journal between the flywheel and the adjacent crank, and two others failed similarly very shortly afterwards. In the fourth case, the engineer in charge had noticed an undesirable roughness in running at 150 r.p.m., and had therefore operated the engine at a lower speed. Examination failed to show any sign of incipient failure, and this set continued to run successfully at the lower speed. All four sets were rebuilt in accordance with the results of investigations, which showed that for the original design 150 r.p.m. was a critical speed, and that a modified construction would avoid the difficulty.

3. Three engines of the airship *Graf Zeppelin* were put out of action during one cruise by crankshaft failure due to torsional vibration. Recurrence of such breakdowns was avoided by inserting a spring coupling between each crankshaft and its propeller, the result being to change the natural frequency of torsional vibration and thus to free the operating speed range

from the critical speed which led to the previous failures.

4. A Diesel-driven d.c. generator used for lighting purposes was found to be unsatisfactory in that it produced objectionable flicker of the lights when running at its designed speed. The trouble did not arise if the running speed was appreciably higher or lower than the nominal one, but there were objections to such departures from the original intention. Calculation showed that the running speed was close to a critical speed for torsional vibration, and although the calculated stresses at this speed did not suggest that there was any danger of mechanical failure, the amplitude of angular vibration at the generator armature had sufficient effect on the electrical system to produce flicker.

The flickering was overcome by fitting an auxiliary flywheel at the tail end of the generator shaft. Although this flywheel was relatively small, its addition changed the vibration characteristics of the system sufficiently to move the troublesome critical speed away from the running speed.

Vibratory Stresses. In designing members which will be subjected to vibrating loads leading to fluctuating stresses of appreciable amplitude it is desirable to take precautions that might safely be ignored under conditions of static loading.

In highly stressed members it is essential to avoid sharp corners or sudden changes of section, because in such regions local stresses are much higher than would be the case in a similarly dimensioned member of uniform cross-section. In ductile materials there is the possibility that local yielding under high stress will relieve stresses at other points to such an extent that danger is avoided. There is, however, no certainty about this, as the endurance limit for reversed stress is well below the yield point even in a ductile material. Materials which are brittle, or tend to be so, e.g. cast iron, phosphor-bronze, high-tensile steel, or case-hardened steel, are particularly susceptible to failure as the result of stress concentration.

Where a change of section is unavoidable, it should be made as gradually as possible. Where a distinct step is necessary, as, for example, on a shaft to provide location for some member carried by the shaft, a fillet of the largest convenient radius should be used. As an illustration of the magnitude of the effect of a sudden change in section it may be mentioned that if the radius of the fillet is less than one-fortieth of the smaller shaft diameter, the highest local torsional stress is more than twice that in the smaller shaft at sections distant from the step.

Oil holes in shafts are preferably radial or axial. The stress-concentration factor, even for a radial hole, is theoretically as much as 4, i.e. the highest local stress near the hole is four times that in the undrilled shaft; but diagonally drilled holes produce appreciably higher stress concentrations than this. A diagonal hole passing close to the surface of the shaft at a change of section is particularly dangerous.

Where alternating stresses are high, the avoidance of stress-raising formations is specially important. In some cases, e.g. aero-engine parts, even scratches and machining marks are prohibited, owing to the observed tendency of cracks to start from such imperfections of surface finish.

APPENDIX

DETERMINATION OF STIFFNESS

TABLE I gives formulae from which may be derived the stiffnesses of certain elements of machine construction. The quantity δ or θ is the linear or angular displacement of the point B relatively to A, associated with the application of unit force or torque at the point indicated by the arrow. The stiffness is the reciprocal of δ or θ.

In the table, Q denotes the second moment of area of the section of the stressed member about an axis passing through the centre of gravity of the section and lying in the section in a position perpendicular to the deflecting force.

In the table, J denotes the second moment of area of the section of the stressed member about an axis passing through the centre of gravity of the section and lying perpendicular to it.

The expressions for Q and J may be derived from Table II by substituting the area of the section for W and taking the section to be a solid of zero thickness.

E = Stretch modulus (Young's Modulus) of the material.

C = Modulus of rigidity of the material.

A = Cross-sectional area.

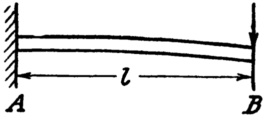
For a steel shaft composed of lengths l_1, l_2, l_3, etc., having external and internal diameters D_1, d_1, D_2, d_2, etc., torsional stiffness (torque required to produce an angle of twist of one radian) is

$$\frac{1{\cdot}16 \times 10^6}{\sum\left(\frac{l}{D^4 - d^4}\right)} \text{ lb. in.}$$

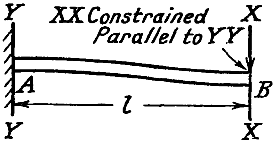
For a keyed connexion to a hub the displacement of the effective junction-plane in the direction of the hub is equal to half the diameter of the shaft.

TABLE I

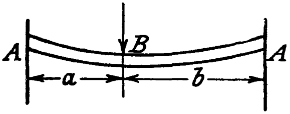
SPECIFIC DISPLACEMENTS



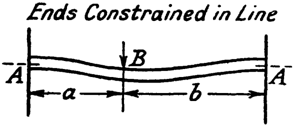
$$\delta = \frac{l^3}{3EQ} + \frac{l}{CA}$$



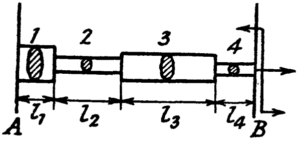
$$\delta = \frac{l^3}{12EQ} + \frac{l}{CA}$$



$$\delta = \frac{1}{3EQ}\,\frac{a^2b^2}{(a+b)} + \frac{1}{CA}\,\frac{ab}{(a+b)}$$



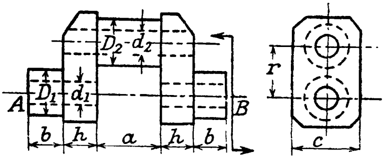
$$\delta = \frac{1}{3EQ}\,\frac{a^3b^3}{(a+b)^3} + \frac{1}{CA}\,\frac{ab}{(a+b)}$$



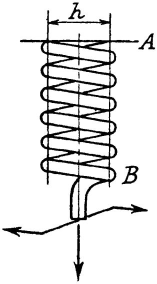
$$\delta = \frac{1}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \frac{l_4}{A_4}\right]$$

$$\theta = \frac{1}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3} + \frac{l_4}{J_4}\right]$$

NOTE 1. The values of l used here differ from the actual lengths by amounts depending on the relative diameters of adjacent parts of the shaft. The effective junction plane is displaced from the actual plane in the direction of the larger diameter by a distance x, where $x = (D_2/8)\{1 - (D_2/D_1)\}^{0 \cdot 4}$, where D_2 is the smaller diameter and D_1 the larger.



$$\theta = \frac{10{\cdot}2}{C}\left[\frac{2b + 0{\cdot}4h}{D_1^4 - d_1^4} + \frac{1{\cdot}1a}{D_2^4 - d_2^4} + \frac{1{\cdot}28r}{hc^3}\right]$$

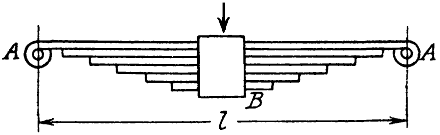


Wire diameter $= d$

Number of coils $= q$

$\delta = 8h^3q/Cd^4$

$\theta = 64hq/Ed^4$



Number of plates $= q$

Width ,, ,, $= u$

Thickness ,, ,, $= j$

$$\delta = \frac{3}{8}\,\frac{l^3}{Equj^3}$$

Note 2. This formula applies if the lengths of the plates of the spring are such that the total moment of resistance at any point is proportional to the distance of that point from the nearer eye of the spring.

Effect of Size of Load. If the mass carried by a rotating shaft has a considerable moment of inertia about an axis perpendicular to the shaft, the effect is to raise the whirling speed. If the mass is carried between bearings, this modification does not exceed about 4 per cent. In the case of an overhung mass the effect may be considerable.

If W = Weight of overhung mass,
l = Distance from nearest bearing,
V = Radius of gyration about an axis intersecting shaft axis perpendicularly,
d = Diameter of shaft, and
N = Whirling speed,

then—

$$N = 227\,000\, d^2 \sqrt{\left[\frac{1}{Wl^3}\left(12{\cdot}2 - \frac{4{\cdot}5}{(V^2/l^2) + 0{\cdot}49}\right)\right]} \text{ r.p.m.}$$

DETERMINATION OF MOMENTS OF INERTIA

Table II gives in each case the formula for the moment of inertia of the body about certain axes passing through the centre of gravity. Other cases for which the expressions for moment of inertia are easily derivable are these—

1. To determine the moment of inertia about an axis parallel with XX (or YY or ZZ) and distant e from XX (or YY or ZZ), modify the expression for moment of inertia about XX (or YY or ZZ) by adding We^2.

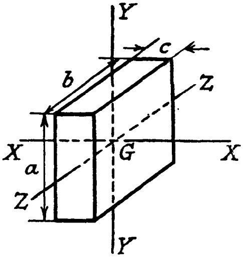
2. The moment of inertia of a hollow body about any axis is the difference between the moments of inertia about that axis of the solid bodies corresponding to its outer and inner surfaces.

3. The moment of inertia of a frustum of cone or pyramid about an axis passing through its imaginary vertex is the difference between the moments of inertia about that axis of the cones or pyramids between which the frustum is the difference. The moment of inertia

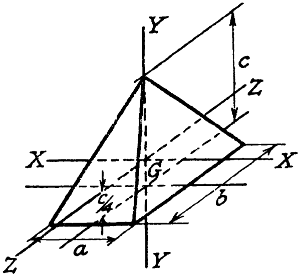
about a parallel axis through the centre of gravity is obtained by subtraction, using (1), and that about any other parallel axis follows from addition, again using (1).

TABLE II

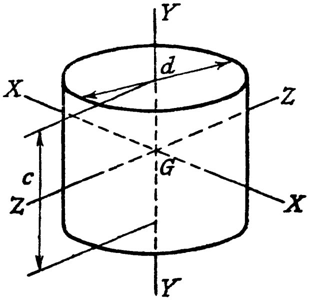
MOMENTS OF INERTIA



$$W = z(abc)$$
$$I_{XX} = W(a^2 + b^2)/12$$
$$I_{YY} = W(b^2 + c^2)/12$$
$$I_{ZZ} = W(c^2 + a^2)/12$$

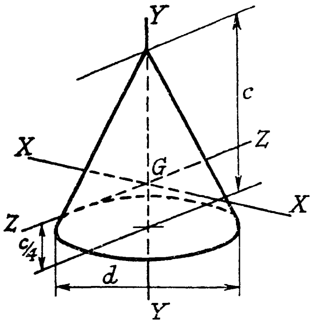


$$W = z(abc/3)$$
$$I_{XX} = \frac{W}{80}[3c^2 + 4b^2]$$
$$I_{YY} = \frac{W}{20}[a^2 + b^2]$$
$$I_{ZZ} = \frac{W}{80}[3c^2 + 4a^2]$$



$$W = \frac{\pi}{4} z d^2 c$$
$$I_{XX} = W\left(\frac{c^2}{12} + \frac{d^2}{16}\right)$$
$$I_{YY} = Wd^2/8$$
$$I_{ZZ} = I_{XX}$$

TABLE II (CONTD.)

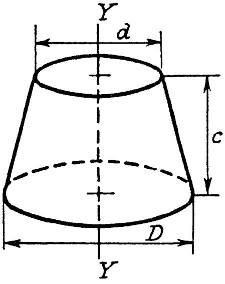


$$W = \frac{\pi}{12} z d^2 c$$

$$I_{XX} = W\frac{3}{80}(c^2 + d^2)$$

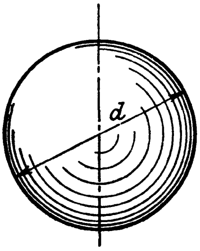
$$I_{YY} = W\frac{3}{40}d^2$$

$$I_{ZZ} = I_{XX}$$



$$W = \frac{\pi}{12} zc(D^2 + Dd + d^2)$$

$$I_{YY} = W\frac{3}{40}\left[\frac{D^5 - d^5}{D^3 - d^3}\right]$$

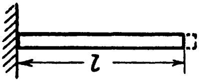
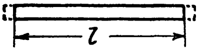
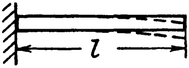
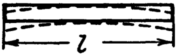
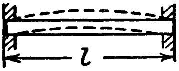
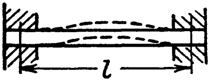
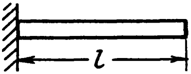
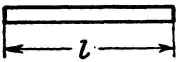


$$W = \frac{\pi}{6} z d^3$$

$$I_{dia} = Wd^2/10$$

TABLE III

NATURAL FREQUENCIES OF STEEL RODS, DIAMETER d, LOADED ONLY BY OWN MASS

Longitudinal		$N = (6{\cdot}1 \times 10^6/l)$ $\times\ 3,\ \times\ 5,\ \times\ 7$, etc.
		$N = (12{\cdot}2 \times 10^6/l)$ $\times\ 2,\ \times\ 3,\ \times\ 4$, etc. These apply to any uniform section. For a material other than steel, multiply by $\frac{1}{10\,350}\sqrt{\frac{E}{z}}$
Transverse		$N = 1{\cdot}71 \times 10^6(d/l^2) \times 10{\cdot}7,\ \times\ 30$
		$N = 10{\cdot}9 \times 10^6(d/l^2) \times 4,\ \times\ 9$, etc.
		$N = 4{\cdot}8 \times 10^6(d/l^2) \times 4,\ \times\ 9$, etc.
		$N = 10{\cdot}9 \times 10^6(d/l^2)$ All these apply to uniform solid circular sections. For other sections, replace d by $4\sqrt{(Q/A)}$, where Q applies to an axis perpendicular to plane of bending. For a material other than steel, multiply by $\frac{1}{10\,350}\sqrt{\frac{E}{z}}$
Torsional		$N = (1{\cdot}91 \times 10^6/l)$ $\times\ 3,\ \times\ 5,\ \times\ 7$, etc.
		$N = (3{\cdot}82 \times 10^6/l)$ $\times\ 2,\ \times\ 3,\ \times\ 4$, etc. These apply to any uniform section. For a material other than steel, multiply by $\frac{1}{6\,500}\sqrt{\frac{C}{z}}$.

The formula given in each case applies to the lowest natural frequency (single-node vibration form). In certain cases multiplying factors are given to indicate the natural frequencies of vibration with 2, 3, 4, etc., nodes. For example, in the first case, the natural frequency for the three-node vibration form is five times that for the single-node form.

TABLE IV

FREQUENCY FACTORS FOR TORSIONAL VIBRATION (p. 45)

Number of Equal Rotors	A	B	C
4	0·172	0·42	– 0·0510
5	0·109	0·344	– 0·0278
6	0·0753	0·294	– 0·0171
7	0·0548	0·256	– 0·0112
8	0·0415	0·222	– 0·0076
9	0·0329	0·204	– 0·0056
10	0·0264	0·182	– 0·0041

TABLE V

TYPICAL VALUES OF HARMONIC COMPONENTS OF TORQUE EXERTED ON CRANK OF FOUR-STROKE COMPRESSION-IGNITION ENGINE AT M.E.P. OF 100 LB./IN.^{2}

Order of Harmonic Component	$\dfrac{\text{Amplitude of Harmonic Component}}{\text{Mean Value of Total Torque at M.E.P. 100 lb./in.}^2}$
½	1·8
1	3·2
1½	2·6
2	2·0
2½	1·5
3	1·2
3½	0·95
4	0·77
4½	0·65
5	0·55
5½	0·45
6	0·37
6½	0·32
7	0·27
7½	0·22
8	0·17

TABLE VI

SOME PHYSICAL PROPERTIES OF CERTAIN MATERIALS

Material	Density z (lb./in.^{3})	E lb./in.^{2}	C lb./in.^{2}	Allowable Vibratory Stress (lb./in.^{2})		Damping Factor H	
				Direct	Shear	Direct	Shear
Rubber	0·04	400	100	± 150*	± 35	0·3	0·35
Cork	0·01	1 500	—	± 25*	—	0·4	—
Phosphor-bronze . . .	0·30	15×10^6	6×10^6	± 5 000	± 4 000	0·004	0·006
Cast iron	0·26	17×10^6	7×10^6	± 3 000	± 3 000	0·007	0·009
Steel 0·3% C . . .	0·28	30×10^6	$11{\cdot}8 \times 10^6$	± 9 000	± 7 000	0·005	0·007
„ 0·5% C . . .	„	„	„	± 10 000	± 8 000	„	„
„ 3·5% Ni . . .	„	„	„	± 11 500	± 9 000	„	„
„ 3% Ni, 1% Cr . .	„	„	„	± 14 000	± 11 000	„	„

* Compression only.

The figures given for rubber and cork are for guidance only. They are liable to considerable variation between different qualities of material.

For vibration insulation, rubber should not be used for frequencies below about 200 cycles/min. or cork below 400 cycles/min.

SOME GENERAL FORMULAE APPLYING TO SIMPLE HARMONIC VIBRATION

Amplitude = λ (inches or radians)

Weight = W lb.

Moment of inertia = I lb. in.^{2}

Frequency = N cycles/min.

$$\left.\begin{array}{ll}\text{Maximum force} & = \dfrac{W\lambda N^2}{35\,200}\text{ lb. in.}\\[2ex] \quad\text{or maximum torque} & = \dfrac{I\lambda N^2}{35\,200}\text{ lb. in.}\end{array}\right\}$$

$$\left.\begin{array}{ll}\text{Vibration energy} & = \dfrac{W\lambda^2 N^2}{70\,400}\text{ lb. in.}\\[2ex] & \text{or } \dfrac{I\lambda^2 N^2}{70\,400}\text{ lb. in.}\end{array}\right\}$$

Maximum velocity = $\dfrac{\lambda N}{9{\cdot}55}$ in. or radians/sec.

Energy dissipated per cycle = $\pi n S\lambda^2$

where n = frequency constant, and damping resistance (force or torque) is equal to S times the vibration velocity in inches or radians per second.

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